OPERATIONAL RISK MEASUREMENT

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Abstract: Operational risk has been great challenges of commercial banks and financial institutions since losses caused by operational risk have been significantly greater than ever before. In order to control and manage operational risk beter, clear awareness of the operational risk conduction mechanism is helpful to further understand the whole dynamic process of the risks affecting the banks. At large financial institutions, operational risk is gaining the same importance as market and credit risk in the capital calculation. Although scenario analysis is an important tool for financial risk measurement, its use in the measurement of operational risk capital has been arbitrary and often inaccurate. The fact that performant strategies of the financial institutions have programmes and management procedures for the banking risks, which have as main objective to minimize the probability of risk generation and the bank's potential exposure,we wants to present the operational risk measurement. Therefore, in this paper we presents the approach assumed by a financial institution with a precise purpose: the quantification of the minimum capital requirements of the operational risk.We implemented the model using discrete-event and Monte Carlo simulation techniques. Results from the simulation are evaluated to show how specific parameter changes affect the level of operational risk exposure for this company.

Keywords: discrete-event simulation, financial regulation, Monte Carlo simulation, risk management, operational risk

1. Introduction

This paper focuses on the issues related to managing risk at the firm level as well as ways to improve productivity and performance. Evaluating Value at Risk Methodologies: Accuracy versus Computational Time.

The paper also presents a new method for using order statistics to create confidence intervals for the errors and errors as a percent of true value at risk for each VAR method. This makes it possible to easily interpret the implications of VAR errors for the size of shortfalls or surpluses in a firm's risk based capital. Value at risk is usually defined as the largest loss in portfolio value that would be expected to occur due to changes in market prices over a given period of time in all but a small percentage of circumstances. This percentage is referred to as the confidence level for the value at risk measure. Value at risk's prominence has grown because of its conceptual simplicity and flexibility. It can be used to measure the risk of an individual instrument, or the risk of an entire portfolio. The purpose of this paper is to examine the tradeoffs between accuracy and computational time The need to examine accuracy is especially poignant since the most recent Basle market risk capital proposal sets capital requirements based on VAR estimates from a firms own internal VAR models. Tradeoffs between accuracy and computational time are most acute when portfolios contain large holdings of instruments whose payoffs are nonlinear in the underlying risk factors because VAR is the most difficult to compute for these portfolios.

Basel I , **Basel II**

The Basel Committee on Banking Supervision, tried to develop a common framework, and this effort led to the Basel I Accord in July 1988. Basel I Capital Accord represented the first key coordinated effort by regulatorsto define a minimum and partially risk-sensitive level of capital for international banks.

The cornerstone of the accord was the *Cooke ratio* (named for its promoter), requiring that the ratio between regulatory capital (RC) and the sum of risk-weighted bank assetsbe equal to or greater than 8%:

$$\frac{RC}{\sum_{i=1}^{n} A_i \times RW_i} \geq 8\%$$

where Ai is a generic asset and RWi represents the corresponding risk weight, based on credit risk only.

This was the fi rst move toward greater use of internal.

In June 2004, the Basel Committee released the ‘Revised Framework for the International Convergence of Capital Measurement and Capital Standards’. allowing “internationally active” banks to calculate regulatory capital using their own internal models – so called AMA (Advanced Measurement Approaches). Operational Risk Management (ORM). any system developed and implemented by a bank must be “credible and appropriate”, “well reasoned”, “well documented” and “transparent and accessible”. These questions are far from trivial. Banks are beginning to invest considerable sums of money and effort in developing the ORM systems necessary for Basel II. The Basel committee specified only that AMA models must be based on a 99.9th percentile confidence interval of a distribution constructed from internal and external loss data.

Methods for Measuring Value at Risk.

Value at risk (VaR) as the maximum potential loss that a business unit or a position can generate in a given time horizon within a defi ned percentage of potential scenarios (the so-called *confi dence level*), where extremely adverse scenarios are excluded.In formal terms, if V is the value of a given portfolio, $V_0$is its initial value and $1 - \alpha$ is the desired confidence level (e.g., 99%), then

$VaR_{1-\alpha}$(i.e., the amount off loss that can be exceeded only in a percentage of potential cases equal to α) is the amount such that $\Pr(V - V_0 < -VaR_{1-\alpha}) = \alpha$ or alternatively, since $V_0 - V$ represents a loss whenever $V < V_0$, $\Pr(V_0 - V > VaR_{1-\alpha} = \alpha)$. The daily volatility is calculated as the standard deviation $\sigma$of the daily log return R of a given saaet $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(R_i - \bar{R})^2}{n-1}}$ where R_i is the daily log return on day, $\bar{R}$ is the average daily return over the sample period and n is the number of daily returns in the sample. If the return distribution is normal (or Gaussian), then he would be able to derive VaR for his position in a very straightforward manner. The Gaussian distribution is in fact defined by only two parameters (its mean μ and its standard deviation σ), and its distribution function is $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}$. It can easily be shown that the probability of extracting a value in the range $(\mu - k\sigma; \mu + k\sigma)$centered on the mean $\mu$and whose half-size is a multiple k of the standard deviation depends only on the multiple k. In fact, such probability is equal to

$$Pr\{\mu - k\sigma < x < \mu + k\sigma\} = \int_{\mu-k\sigma}^{\mu+k\sigma} f(x)dx = \int_{\mu-k\sigma}^{\mu+k\sigma} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2} dx.$$

By substituting $z = \frac{x-\mu}{\sigma}$ one can easily obtain

$$Pr\{\mu - k\sigma < x < \mu + k\sigma\} = \int_{\mu-k\sigma}^{\mu+k\sigma} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2} = \int_{-k}^{+k} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz.$$

We will measure value at risk over a fixed span of time (normalized to one period) in which positions are assumed to remain fixed many firms set this span of time to be one day. This amount of time is long enough for risk over this horizon to be worth measuring, and it is short enough that the fixed position may reasonably approximate the firm's one-day or overnight position for risk management purposes.

Denote $V(P_t, X_t, t)$ as the value of portofolio V at time t with instruments X_t and instrument prices P_t and denote $\Delta V(P_{t+1} - P_t, X_t, t)$ as the change in portfolio value between period t and t+1. The cumulative density function of $\Delta V(P_{t+1} - P_t, X_t, t)$ conditional on X_t and t information I_t is:

$$G(k, I_t, X_t) = Probability(\Delta V(P_{t+1} - P_t, X_t) \leq (k|I_t)).$$

This allows the inverse cumulative density function for $\Delta V(P_{t+1} - P_t, X_t, t)$ to be defined as:

$$G^{-1}(u, I_t, X_{t},) = inf\{k: G(k, I_t, X_t) = u\}$$

Value at risk for confidence level u is defined on terms of the inverse cumulative density function of ΔV:

$$VAR(u, I_t, X_t) = G^{-1}(u, I_t, X_t).$$

In words, value at risk at confidence level u is the largest loss that is expected to occur except for a set of circumstances with probability u. This is equivalent to defining value at risk at confidence level u as the u'th quantile of the ditribution of $\Delta V(P_{t+1} - P_t, X_t)$ given I_t.

The definition of VAR highlights its dependence on the function G^{-1} which is a conditional function og the instruments X_t and the information se I_t.

Value at Risk methods attempt to implicitly or explicitly make inferences about $G^{-1}(u, I_t, X_t)$ in a in a neighborhood near confidence level u. In a large portfolio G^{-1}depends on the joint distribucion of potentially tens of thousands of different instruments. This makes it necessary to make simplifying assumptions in order to compute VAR; these assumptions usually take three forms. First, the dimension of the problem is reduced by assuming that the price of the instruments depend on a vector of factors f that are the primary determinants of changes in portfolio value.This allows changes in portofolio value to be expressed as $\Delta V(\epsilon_{t+1}, X_t, t)$ where $\epsilon_{t+1} = f_{t+1} - f_t$. Second, $\Delta V(\epsilon_{t+1}, X_t, t)$ is usually approximated instead of being calculated explicitly. Finally, convenient functional forms are often assumed for the distribution of ϵ_{t+1}.

Each of these simplifying assumptions is likely to introduce errors in the VAR estimates. The first assumption induces errors if an incorrect or incomplete set of factors is chosen; the second assumption introduces approximation error; and the third assumption introduces error if the wrong distribution of ϵ_{t+1} is chosen. The first and third assumption introduces error if the wrong distribution of ϵ_{t+1} is chosen.

methods use Monte-Carlo simulation to compute value at risk. Their advantage is they are capable of producing very accurate estimates of value- at-risk, however, these methods are not analytically tractable, and they are very time and computer intensive.

Monte-carlo methods

Monte Carlo Simulation includes any statistical sampling technique used for approximate quantitative problems solutions.

A model or a real system or situation is developed and this model contains certain variables. These variables have different probable values, represented by a probability distribution function values for each variable. monte carlo method simulates the complete system many times (hundreds or even thousands times), each time randomly selects a value for each variable from occasional its probability distribution. The result is a probability distribution of the total value of the system calculated by the model.

Monte carlo is a technique that simulates cost estimates or the timing of the fulfillment of specific works many times using input values selected at random from the probability distributions of possible costs or the likely duration to clculate a distribution of possible total cost of the project or full time to the competion of the project possible.

The next set of approaches for calculating value at risk are based on Monte-Carlo simulation.

Monte Carlo simulation works by using a series of random draws of the factor shocks (ϵ_{t+1}). These shocks combined with some approximation method are used to generate a random series of changes in portofolio value (ΔV). The empirical cumulative density function of the changes in portofolio value, $\hat{G}$, is then used as a proxy for G, and $\widehat{G^{-1}}(u)$ is the corresponding estimate of VAR at cofidence level u.

The Monte-Carlo methodologies considered here allow for two methods for approximating changes in protfolio value and methods of parameterizing the distribution of ϵ_{t+1}. These approaches can be combined.

The first approximation method is the **full Monte-Carlo** method. This method uses exact pricing for each Monte-Carlo draw, thus eliminating errors from approximations to ΔV. Since this method involves no approximation error, $\hat{G}$, the full Monte-Carlo estimate of G converges to G in probability as the sample size grows provided the distributional assumptions are correct. Consequently, $\widehat{G^{-1}}(u)$ converges in probability to true value at risk at confidence level u. Because this approach produces good estimates of VAR for large sample sizes, the full Monte-Carlo estimates are a good baseline against which to compare other methods of computing value at risk. The downside of the full Monte-Carlo approach is that it tends to be very time-consuming, especially if analytic solutions for some assets prices don't exist.

The second approximation method is the grid Monte-Carlo approach. In this method a grid of realizations for ϵ_{t+1} (for N factors, this would involve an N dimensional grid) is created and the change in portfolio value is calculated exactly for each node of the grid. To make approximations using the grid, for each Monte-Carlo draw, the factor shocks should lie somewhere on the grid, and changes in portfolio value for these shocks can be estimated by interpolating from changes in portfolio value at nearby nodes.

Grid-Monte-Carlo Methods suffer from a curse of dimensionality problem since the number of grid points grows exponentially with the number of factors. To avoid the dimensionality problem, I model the change in the value of an instrument by using a low order grid captures the behavior of factors that combined with a first order Taylor series. The grid generate relatively nonlinear changes in instrument value, while the first order Taylor series captures the effects of factors that generate less nonlinear changes.

Suppose H is a financial instrument whose value depends on two factors, f_1, and f_2. In addition suppose H is highly nonlinear in f_1 and nearly linear in f_2. Define ϵ_1 as the change in f_1 over the next period and ϵ_2 as the change in f_2. Then the change in Hbetween today and tomorrow is an implicit function of the change in the factors and can thus be written as $\Delta H(\epsilon_1, \epsilon_2)$. If ΔH was approximated on a grid using 10 values of ϵ_1 and ten values of ϵ_2 then the grid would contain 100 points and just computing the points on the grid would be computationally intensive. Below, we model changes in value due to changes in ϵ_1 on a grid and model changes in value due to ϵ_2 using a first order Taylor series. More specifically:

$$\begin{aligned}\Delta H(\epsilon_1, \epsilon_2) &= \Delta H(\epsilon_1, 0) + [\Delta H(\epsilon_1, \epsilon_2) - \Delta H(\epsilon_1, 0)].\\ &\approx \Delta H(\epsilon_1, 0) + \Delta H_2(\epsilon_1, 0)\epsilon_2\\ &\approx \Delta H(\epsilon_1, 0) + \Delta H_2(0,0)\epsilon_2\end{aligned}$$

Where $\Delta H_2(\epsilon_1, 0) = \frac{\partial}{\partial \epsilon_2} \Delta H(\epsilon_1, \epsilon_2)\Big|_{\epsilon_2=0}$.

Confidence Intervals from Monte-Carlo Estimates.

To construct a 95% confidence interval for the p'th percentile of G using the results from

Monte-Carlo simulation, it suffices to solve for an r and s such that:

$$\sum_{i=r}^{s-1} \binom{N}{i} p^i (1-p)^{N-i} \geq .95$$

and such that:

$$\sum_{i=r+1}^{s-1} \binom{N}{i} p^i (1-p)^{N-i} \leq .95$$

Then the order statistics X(r) and *X(s)* from the Monte-Carlo simulation are the bounds

forthe confidence interval.

Measuring the Accuracy of VAR Estimates.

Estimates of VAR are generally not equal to true VAR, and thus should ideally be accompanied by some measure of estimator quality such as statistical confidence intervals or a standard error estimate. This is typically not done for delta and delta-gamma based estimates of VAR since there is no natural method for computing a standard error or constructing a confidence interval.

In Monte-Carlo estimation of VAR, anecdotal evidence suggests that error bounds are typically nor is controlled somewhat by making Monte-Carlo draws until the VAR estimates do not change significantly in response to additional draws. This procedure may be inappropriate for VAR calculation since value at risk is likely to depend on extremal draws which are made infrequently. Put

differently, value at risk estimates may change little in response to additional Monte-Carlo draws even if the value at risk estimates are poor.
Although error bounds are typically not provided for full Monte-Carlo estimates of VAR, it is not difficult to use the empirical distribution from a sample size of N Monte-Carlo draws to form confidence intervals for Monte-Carlo value at risk estimates (we will form 95% confidence intervals). Given the width of the interval, it can be determined whether the sample of Monte-Carlo draws should be increased to provide better estimates of VAR. The confidence intervals that we will discuss have the desireable properties that they are nonparametric (i.e. they are valid for any continuous distribution function G), based on finite sample theory, and are simple to compute. There is one important requirement: the draws from the distribution of ϵ_{t+1} must be independently and identically distributed. The confidence intervals for the Monte-Carlo estimates of value at risk can also be used to construct confidence intervals for the error and percentage error from computing VAR by other methods. These confidence intervals are extremely useful for evaluating the accuracy of both Monte-Carlo methods and any other method of computing VAR.

The use of confidence intervals should also improve on current practices of measuring VAR error. The error from a VAR estimate is often calculated as the difference between the estimate and a Monte-Carlo estimate. This difference is an incomplete characterization of the VAR error since the montecarlo estimate is itself measured imperfectly. The confidence intervals give a more complete picture of the VAR error because they take the errors from the Monte-Carlo into account.Table 1 provides information on how to construct 95% confidence intervals for monte carlo estimates of VAR for VAR confidence levels of one and five percent. To illustrate the use of the table, suppose one makes 100 iid Monte-Carlo draws and wants to construct a 95% confidence two interval for value at risk at different ways. Then, the upper right column of table 1 shows that the portfolio loss from the Monte-Carlo simulations and the 10th largest portfolio loss 95% confidence interval for true value at risk. The parentheses below these figures the confidence bounds in terms of percentiles of the Monte-Carlo distribution. Hence, the first percentile and 10th percentile of the Monte-Carlo loss distribution bound the 5th percentile of the true loss distribution with 95% confidence when 100 draws are made.

Table 1

NumberofDraws	Value at Risk CofidenceLevel	Value at Risk CofidenceLevel
	1%	5%
	LowerBoundUpperBound	LowerBoundUpperBound

100	- -	- -	1 (1%)	10 (10%)
300	1 (0.33%)	11 (3.67%)	8 (2.7%)	23 (7.7%)
500	1 (0.2%)	10 (2.0%)	15 (3.0%)	35 (7.0%)
1,000	4 (0.4%)	17 (1.7%)	37 (3.7%)	64 (6.4%)
10,000	81 (0.81%)	120 (1.2%)	457 (4.57%)	544 (5.44%)
50,000	456 (0.912%)	545 (1.09%)	2404 (4.81%)	2597 (5.19%)
100,000	938 (0.938%)	1063 (1.063%)	4865 (4.865%)	5136 (5.136%)
250,000	2402 (0.9608%)	2599 (1.0396%)	12,286 (4.9144%)	12,715 (5.086%)
500,000	4862 (0.9742%)	5139 (1.0278%)	24,698 (4.9396%)	25,303 (5.0606%)
1,000,000	9805 (0.9805%)	10196 (1.0196%)	49,573 (4.9573%)	50,428 (5.0428%)

Table 1 provides upper and lower bounds for larger numbers of draws and for VAR confidence levels of l% and 5%. As one would expect, the table shows that as the number of draws increase the bounds, measured in terms of percentiles of the Monte-Carlo distribution, tighten, so that with 10,000 draws, the 4.57th and 5.44th percentile of the Monte-Carlo distribution bound the 5th percentile of the true distribution with 95% confidence.

Table 1 can also be used to construct confidence intervals for the error and percentage errors from computing VAR using Monte-Carlo or other methods.

Computational Considerations.

Risk managers often indicate that complexities of various VAR methods limit their usefulness for daily VAR computation. The purpose of this section is to investigate and provide a very rough framework for categorizing the complexity of various VAR calculations.

Computations involved in approximating the value of the portfolio and its derivatives. This type can be further segregated into two groups. The first are *complex* computations that involve the pricing of an instrument or the computation of a partial derivative such as $\delta or \Gamma$.

I also make assumptions about the number of complex computations that are required to price an instrument. In particular, I will assume that the pricing of an instrument requires a single complex computation no matter how the instrument is actually priced.

The amount of time that is required to calculate VAR depends on the number and complexity of the instruments in the portfolio, the method that is used to calculate VAR, and the amount of parallelism in the firms computing structure.

Let V denote a portfolio whose value is sensitive to N factors, and contains a total of I different instruments. An instrument's complexity depends on whether its price has a closed form solution, and on the number of factors used to price the instrument. For purposes of simplicity, I will assume that all prices have closed form solutions, or that no these extreme cases. Instruments have closed form solutions, and present results for both of To model the other dimension of complexity, let I_n,denote the number of different instruments whose value is sensitive to n factors. It follows that the number of instruments in the portofolio is $I = \sum_{n=1}^{N} I_n$.

The number of complex computations required for the delta approach is:

$$Delta = \begin{cases} \sum_{n}^{N} I_n n if analytical calculation \\ \sum_{n}^{N} I_n 2n\, if numerical calculations \end{cases}$$

Similarly, the number of complex computations required for all of the delta-gamma approaches is equal to:

$$Delta - Gamma = \begin{cases} \sum_{n=1}^{N} \frac{I_n(n^2+3n)}{2} if analytical calculations \\ \sum_{n=1}^{N} I_n(2n^2+3n)\, if numerical calculations \end{cases}$$

The grid-Monte-Carlo approach prices the portfolio using exact valuation at a grid of factor values and prices the portfolio at points between grid values using some interpolation technique (such is linear). For simplicity, i will assume that the grid is computed for k realizations of each factor. This then implies an instrument that is sensitive to n factors will need to be repriced at k^n grid points. This implies that the number of complex computations that is required is: $GridMonte - Carlo = \sum_{n=1}^{N} I_n k^n$

The number of computations required for the grid Monte-Carlo approach is growing exponentially in n. For even moderate sizes of n, the number of computations required to compute VAR using grid-Monte-Carlo is very large. The number of complex computations required for one instrument using the modified grid Monte-Carlo approach is $k^m + n - m$ if the firs derivatives in the taylor series are computed analytically, and $k^m + 2(n - m)$ if they are computed numerically.

The number of complex computations required in the modified grid Monte-Carlo approach with a grid consisting of k realizations per factor on the grid is:

$$ModifiedGridMonteCarlo = \begin{cases} \sum_{n=1}^{N}\sum_{m=1}^{n} I_{m,n}[k^m + n - m]\, if analytical \\ \sum_{n=1}^{N}\sum_{m=1}^{n} I_{m,n}[k^m + 2(n - m)]\, if numerical \end{cases}$$

Finally, the number of computations required for the exact pricing Monte-Carlo approach (labelled Full Monte Carlo) is the number of draws made times the number of instruments that are repriced:

$$FullMonteCarlo = NDRAWS \sum_{n=1}^{N} I_n$$

Only one simple computation is required in the delta approach and in most of the deltagamma approaches. The number of simple computations in the grid Monte-Carlo approach is equal to the expected number of linear interpolations that are made.

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