A PROBABILITY APPROACH TO HUBBERT METHOD OF OIL PRODUCTION AND RESERVES ESTIMATION, APLICATIONS IN ALBANIAN OILFIELDS

Robert Kosova.[1], Evgjeni Xhafaj.[2], Daniela Qendraj.[3], Irakli Prifti.[4]

[1,2,3]Department of Mathematics, Faculty of IT, University “A. Moisiu”, Durrës

[4]Department of Earth Sciences, Faculty of Geology and Mining, Politechnic University, Tirana

Abstract: Evaluation of oil reserves in an oilfield is one of the most important tasks in oil industry. While, we can estimate oilfield reserves by using common formulas and methods such as volumetric formula, another method of estimation is produced by studying and analysing the productionlife of the oilfield or its historical production data. The Hubbert model of oil production and Reserves estimation is well established in the oil industri since 1956, when geologist Marion King Hubbert proposed the bell- shape symetrical curve as the model for United States oil past and future production. His prediction of peak-oil of United States confirmed to be right. Hubbert at the time did not present any formula or mathematical model to justify his theory. Many scientifics after him, proposed methods and mathematical models to explain or complete his method; (Aleklett and Campbell, 2003; Bentley, 2002; Brandt 2006; Campbell and Laherrère, 1998; Deffeyes, 2001). In this paper, we will focus on analysing and applying a statistical model, which is very useful to describe oil production history and future, also to estimate oil reserves of some oilfieldsin Albania.

Key words:*Hubbert method, oil production, probability, reserves, estimation.*

Introduction

The concept of "peak oil", (Hubbert 1956, 1962)includes a number of assumptions;

- production will start from zero, there will be an increase, reaching a maximum point or more and then will end in zero again,
- production follows a bell-shaped curve,
- production is symmetric over time (i.e. the decline in production will mirror the increase in production),
- the year of maximum production, or peak year, occurs when the resource is half depleted; production will follow discovery in functional form and with a constant time lag;

- The area under the curve of production and the time axis will provide total oil production or reserves, as time goes to infinity.

Methods and Methodology

Hubbert later developed a mathematical model based on the logistic curve by assuming that production output at a time $t, q\ (t)$ or $dQ\ /\ dt$ could be expressed as a parabolic function of the cumulative production$Q$(t), (Hubbert, 1962, 1982).

$$q(t) = \frac{dQ}{dt} = aQ + bQ^2 \qquad (1)$$

When cumulative production will be equal to total Production $Q_\infty$the production rate $q(t)$will be equal to zero. That means the end of the production because there is no more petrolium left to produce.

Then, from (1) we have;

$$b = -\frac{a}{Q_\infty} \quad (2)$$

From equations (1) and (2), we have;

$$\frac{dQ}{(Q - \frac{Q}{Q_\infty})} = adt \quad (3)$$

Integrating (3) for$t$from $t_0$to ∞, the logistic cumulative oil production will be;

$$Q = \frac{Q_\infty}{[1 + N_0 e^{-a(t-t_0)}]} \quad (4)$$

Where$N_0 = \frac{Q_\infty - Q_0}{Q_\infty}$(5)

Substituting (4) in(1), the production in time t will be:

$$q(t) = \frac{dQ}{dt} = Q_\infty \frac{aN_0 e^{-a(t-t_0)}}{[1+N_0 e^{-a(t-t_0)}]^2} \quad (6)$$

The maximum cumulative production will be;

$$Q_M = \frac{Q_\infty}{2} (7)$$

Maximum production rate is;

$$q_M = a\frac{Q_\infty}{4}, \qquad (8)$$

$$t_M = t_0 + \frac{ln\,(N_0)}{a} \quad (9)$$

In a more simple form and less parameters tha (4) and (6), we can express cumulative production in terms of total production and peak time from by substituting $t_0$from (9) to (4), then;

$$Q = \frac{Q_\infty}{[1+e^{-a(t-t_M)}]} \quad (10)$$

$$q(t) = \frac{dQ}{dt} = \frac{Q_\infty a e^{-a(t-t_M)}}{[1+e^{-a(t-t_M)}]^2} \quad (11)$$

Equation (10) is the production rate function,simmilar to logistic curve created by Verhulst, (1845), with population growth studies. The equation (11) which is the derivate of (10) is the Hubbert curve, figure 1.

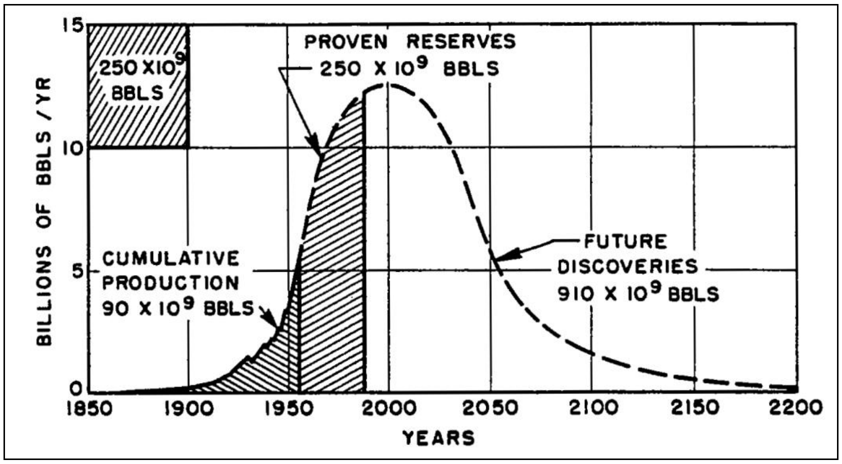


Figure1. The Hubbert curve of oil production of USA, presented in 1956.

Improving Hubbert Model

After Hubbert, many authors, Bartlett (2000), Brandt (2007), Laherrere(1997) also developed different mathematical models to forecast oil production and to estimate ultimate recoverable resources. Laherrere pointed out that Hubbert's model applied for forecasting production works well only in nature's domain, unaffected by political or significant economic interference, and for areas having a large number of fields and independent activity.Hallock(2004)applied a modified bell-shaped curve, with a peak at 60% of ultimate production instead of the 50%. That implies an asymmetric shape ofcurve production and a steeper rate of decline than increase.Exponential models are other possible simple model. Also, Wood et al., (2005) assumed a 2% exponential growth for world oil production, followed by a decline at an R/P ratio of 10%. This decline at a constant R/P of 10 is equivalent to exponential decline of 10% per year.

The Gaussian model

Another mathematical model often used in fitting historical production data is the normal distribution model because of its similarity to Hubbert'scurve. Bartlett (2000) applied successfully the normal distribution model to predict crude oil production for the U.S and the world. This leads to some methodological problem; to justify the use of the Gaussian model. Laherrere(2004), states that the Hubbertmodel works best with large numbers of disaggregated oil producers, based on the central limit theorem (CLT), which is the justification for the application ofGaussian curve in statistical applications. He states that that in the United States there are over 20,000 oil producers acting in random," leading to a Gaussian curve”. Still, there are two problems with using the CLT; first, the CLT traditionallyapplies to data where the independent variable is the measured value of a characteristic and the dependent variable is the number of measurements of that value that are recorded (weight or height of students in a class, etc). Petroleum production measure data (barrels, ton per year, day)is not the traditional dimension for normally distributed phenomena.More, CLT applies only to values distributions that are summed and independent of one another. In reality, production at a given oilfield is determined not only by its reservoir parameters such as area, pressure, porosity, etc, but by the decisions of other factors. These factors are of financial nature, state regulations, local factors, environmental factors, etc. That doesn’t’ mean that, in fact, the production will not follow a normal distribution model.

The Normal model is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \qquad -\infty < x < +\infty \qquad (12)$$

$\mu$is the mean, σ is the standard deviacion,figure 9.

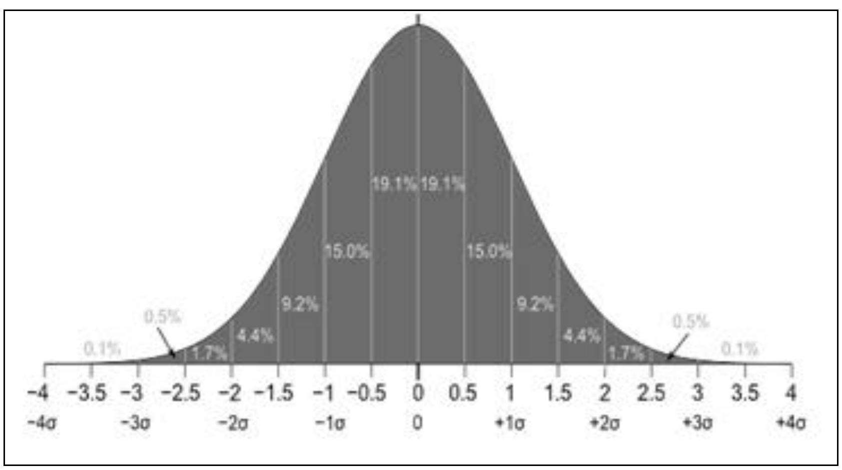


Figure2. Gaussian Distribution, (Normal)

The area under the normal curve is equal to 1. In our case, the area will be equal to total production of the oilfield or Oilfield Reserves. We can rewrite the normal distribution formula with the oilfield parameters, Q_∞, t_M and S_N.

$$q(t) = \frac{Q_\infty}{S_N\sqrt{2\pi}} \cdot e^{-\frac{(t-t_M)^2}{2S_N^2}} \quad (13)$$

$q(t)$ – production rate, Q_∞ – maximum (peak production),

t_M – year of peak, s_N – standard deviation of the production curve.

Case study: Marinza Oilfield- D

MarinzaOilfield started producing oil in 1938. There were about 2400 oil wells working in 2004.(Data: Arshiveof Oil & Gas Institute, Fier,table1, figure 3).

Table1. Oil production 1957- 2004, Marinza- D, (partial data).

Year	Prod. Ton/ y	Cum. Prod. ton	Year	Prod. Ton/ y	Cum.Prod .	Year	Prod. Ton/ y	Cum.Prod .
1957	90000	90000	1974	676510	5447693	1990	168397	12514467
1958	95400	185400	1975	677863	6125556	1991	129536	12644003
1959	101124	286524	1976	679218	6804774	1992	99643	12743646
1960	107191	393715	1977	680577	7485351	1993	97689	12841335
1961	113623	507338	1978	630164	8115515	1994	95774	12937109
1962	120440	627778	1979	583485	8699000	1995	93896	13031005
1963	127667	755445	1980	540264	9239264	1996	78247	13109252

1964	150647	906092	1981	500244	9739508	1997	55890	13165142
1965	177763	1083855	1982	463189	10202697	1998	42993	13208135
1966	209761	1293616	1983	428879	10631576	1999	35827	13243962
1967	268493	1562109	1984	397110	11028686	2000	35720	13279682
1968	332932	1895041	1985	330925	11359611	2001	35613	13315295
1969	412836	2307877	1986	275771	11635382	2002	36325	13351620
1970	511916	2819793	1987	255343	11890725	2003	43591	13395211
1971	634776	3454569	1988	236429	12127154	2004	56668	13451879
1972	636045	4090614	1989	218916	12346070	2005	79335	13531214
1973	680569	4771183						

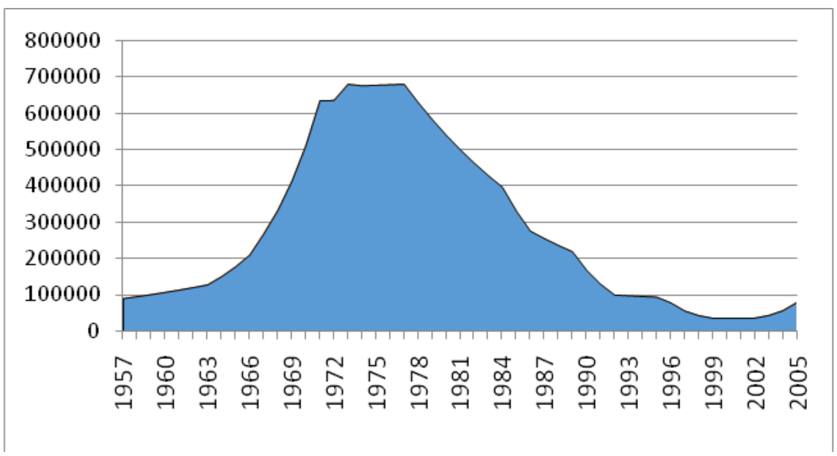


Figure 3. Oil production, Marinzaoilfield.

Hubbert model for Marinza- D oilfield

We used the CurveExpert Pro. software to find the best fit of data. The gaussian model isamong best fit functions, figure 4.

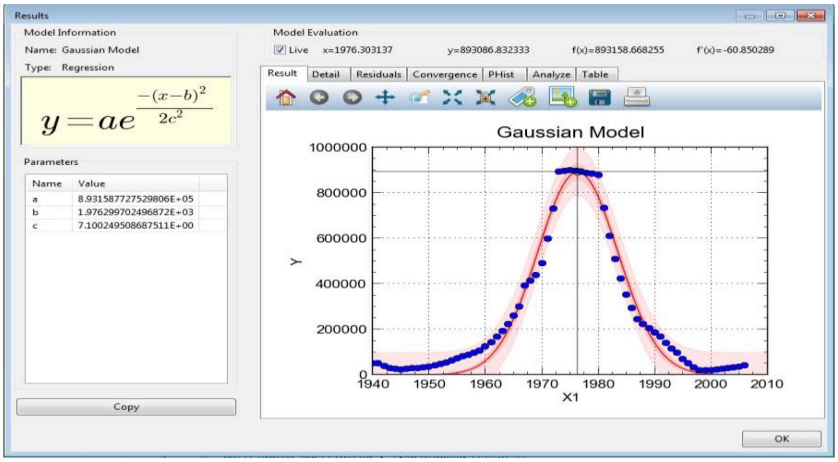
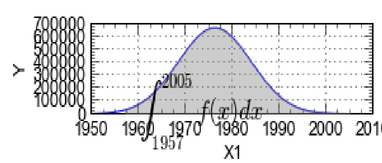
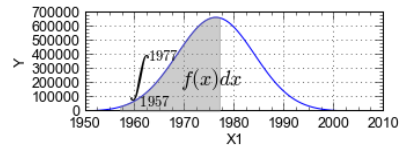
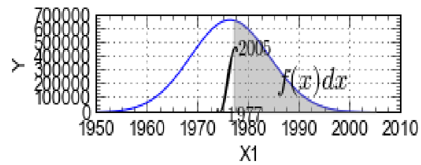


Figure 4. Gausian rmodel fit to production data, Marinza- D.

Results

Table 2. The gaussian regression function results.

Fit Gaussian Model	Parameters	Std. error	Range (95% confidence)
Model data	$a = 666514$	44345.83	(636998, 696029)
Type: Regression,	$b = 1976$	0.154534	(1975, 1977)
Function: $y = ae^{-\frac{(x-b)^2}{2c^2}}$	$c = 7.77$	0.154534	(7.4, 8.2)
Correlation Coeff.	R = .98,		
Dof= 46,			
Area under the curve: 1957- 2005.	12895272 ton		

Area under the curve before peak- oil, 1957- 1976.	6305764 ton	
Area under the curve, after peak- oil,1977- 2005.	5924141 ton	

Theoil production rate function of Marinza- D oilfield is:

$$q(t) = ae^{-\frac{(t-b)^2}{2c^2}}, \quad q(t) = 893158e^{-\frac{(t-1976)^2}{100}} \tag{14}$$

$$q(t) = \frac{Q_\infty}{S_N\sqrt{2\pi}} . e^{-\frac{(t-t_M)^2}{2S_N^2}}, \tag{15}$$

Total Reserves;$Q_\infty = 15895928,$ Peak- oil;$t_M =$ year 1976.

Conclusions

- Hubbert's modelworks well only where applied to a natural domain, unaffected by outside factor such as political or significant economic interferences and to areas having a large number of fields and of indipendent productions;
- Hubbert model gives better results when the production data has been not disturbed by economic and political factors and when the inflection point has been passed.
- Good results are achieved only when the production data series has passed the peak and when there is one single cycle for discovery.Hubbert 's model is a useful tool to predict oil future production and so, to estimate oil reserves of a oilfield.

References

Adam R Brandt. “Testing Hubbert”. Elsevier. Energy Policy. Volume 35, Issue 5, May 2007, Pages 3074–3088.

Adam R. Brandt. “Review of mathematical models of future oil supply: Historical overview and synthesizing critique”. Elsevier. Science Direct. Energy. Volume 35, Issue 9, September 2010, Pages 3958–3974.

Jean Laherrere, "Forecasting production from discovery", ASPO Lisbon May 19–20, 2005.

Marion King Hubbert. “Techniques of prediction with application to the petroleum industry”. In 44th Annual meeting of the American Association of Petroleum Geologists, page 43, Dallas, TX, 1959. Shell Development Company.

Prifti I.,Muska K. (2013). Hydrocarbon occurrences and petroleum geochemistry of Albanian oils. J. Geosci. (Boll. Soc. Geol. It.), Vol. 132, No. 2 (2013), pp. 228-235.