MOTIVATIONS FOR THE CORRECT USE OF ERRORS IN MATHEMATICS IN PRE-UNIVERSITY EDUCATION

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Abstract

Albania comes from a long history of behavioral theories and the error is not yet free from the influence of this theory, even though the curriculum used in pre-university education is based on constructionist approaches. The positive influence of errors in the teaching/learning process has been widely discussed, but in mathematics, there is a lack of real pedagogical plans and differentiated instructional treatment, where there is a tendency to emphasize errors as much as possible. The purpose of this article is to highlight the educational motives of error in the process of enhancing learning in mathematics. Concrete educational planning that makes use of the power of errors is especially important from the perspective of why errors should be studied and what is stimulated by their correct use. The interpretation of mistakes at the metacognitive and non-cognitive levels is of relevance, and we highlight its main parameters, which are self-acceptance, self-control, and self-correction, using a critical and creative approach. The paper provides a solid theoretical framework for the relevance of extensive mistake analysis, where proper interpretation necessitates a variety of pedagogical and didactic treatments that promote learning building. Math instructors and educational institutions at all levels will benefit from this study.

Key words: *error, motivation, mathematics learning.*

Përmbledhje

Shqipëria vjen nga një histori e gjatë e teorive bihevioriste ku gabimi ende nuk është i çliruar nga ndikimi i kësaj teorie, edhe pse kurrikula e përdorur në arsimin parauniversitar bazohet në qasje konstruktiviste. Në këtë punim është diskutuar gjerësisht ndikimi pozitiv i gabimeve në procesin e mësimdhënies/të nxënit, por në rastin e matematikës mungojnë plane reale pedagogjike dhe trajtime të diferencuara didaktike, ku gabimet priren të theksohen sa më shumë. Qëllimi i këtij artikulli është të nxjerrë në pah motivet pedagogjike të gabimeve në procesin e përmirësimit të të nxënit në matematikë. Një projektim didaktik konkret që shfrytëzon fuqinë e gabimeve është veçanërisht i rëndësishëm për sa i përket arsyes pse gabimet duhet të studiohen dhe çfarë stimulohet konkretisht nga përdorimi i tyre i saktë. Interpretimi i gabimeve në nivel metakonjitiv dhe jokonjitiv është i rëndësishëm ku veçohen parametrat kryesor, të cilët janë, vetëpranimi, vetëkontrolli dhe vetëkorrigjimi, duke përdorur qasje kritike dhe krijuese. Ky artikull ofron një kornizë solide teorike për rëndësinë e analizës së hollësishme të gabimeve, interpretimi i saktë i tyre kërkon një shumëllojshmëri trajtimesh pedagogjike dhe didaktike që nxisin ndërtimin e të nxënit. Nga ky studim do të përfitojnë mësuesit e matematikës dhe institucionet arsimore të të gjitha niveleve.

Fjalë kyçe: *gabimet, motivimet, nxënia në matematikë.*

Introduction

According to the radical constructivism notion, mathematical errors provide teaching power (Ingram et al, 2015; McLaren et al, 2012; Montessori, 1970). The constructive approach (Glasersfeld, 1995; Kilpatrick, 1987; Piaget, 1967;) to the learning process is related to their interpretation and analysis (Borasi, 1987; Radatz, 1979; Rushton, 2018; Zan, 2012). A reflection and analysis of the mistakes encourage users' critical and creative thinking, review of the concepts, and a lesson with the idea that belongs to them to discover and recover from the mistake (Lenz et al, 2022). Errors are interpreted and analysed at three levels of sources: cognitive, metacognitive, and noncognitive (Ashcraft, 2016, Keith & Frese, 2005; Zan, 2012). Their detailed interpretation and analysis establishes the chain of subject's competency, i.e., those competences which are in possession and stable and those who should be assessed and obtained. (Borasi, 1987; Grassinger & Dresel, 2017).

Error analysis (Borasi, 1987; Lenz et al, 2022; Radatz, 1979; Rushton, 2018; Zan, 2012;):

- Can provide a tangible stimulus for them to improve in the product - process and learning-outcome directions.
- This is important since the same mistakes in different students are caused by different items, so the source of the error must be identified.
- It should be viewed from the subject's perspective rather than the professionals.
- Errors that are used correctly stimulate (Brodie, 2014; Keith & Frese, 2005; Mollo, 1986; Pankow et al, 2018; Zan, 2012):
- Its acceptance, control, and self-correction.
- Expanding and learning new scenarios.
- Individual and group work, as well as increased interaction between students and between students and teachers.

The definition of error in mathematics

The study of errors is always a "hot issue," as though its origins are in human nature and are fundamental to learning (Ahuja, 2018; Grassinger & Dresel, 2017; Tulis et al, 2017). Error is a normal part of being human, and it is not always a negative trait, as Tagore points out, reminding us that if the door to error is closed, even the truth remains outside (Pepkolaj & Duraj, 2020). There was a discourse about mathematical mistakes in a positive light as early as the seventeenth century.

Maria Montessori refers to him as the "Lord of Error" and emphasizes the need for exact sciences in drawing attention to him, exactly because it is their responsibility to do so. (Brodie, 2014; Ingram et al, 2015; Montessori 2001). A good definition of error is as follows:

- Mathematical errors are the results of specific processes whose nature must be discovered. They are not simply a lack of correct answers or the outcome of unfortunate anomalies.
- From the standpoint of information mechanisms and individual processing, it appears that it is possible to analyze the nature and underlying causes of errors from the standpoint of the individual.

- Error analysis gives a variety of beginning points for research on how students learn mathematics (Radatz, 1979).

Analysis of errors

We use three levels of resources and mental processes to investigate errors (Ahuja, 2018; Ashcraft, 2016; Keith & Frese, 2005; Zan, 2012):

- Cognitive: the process by which a subject preserves, hypothesizes, and generates knowledge.
- Metacognitive: refers to a situation in which the subject is involved in the process of cognitive control and awareness.
- Non-cognitive: in which it is up to the subject's responsibility to acquire knowledge, which is linked to attractive and motivating factors.

Level of cognition

Why is it essential to analyse errors (Borasi, 1987; McLaren et al, 2015; Radatz, 1979; Rushton, 2018)?

When these types of mistakes are used constructively (Brousseau, 1981; Glasersfeld, 1995; Große & Renkl, 2007; Tulis et al, 2017; Vosniadow & Verschaffel, 2004) they can stimulate analytical thinking about the competence that has occurred and what will be achieved (Figure 1), which aims to (Borasi, 1994; Hill et al, 2008):

- A better analogy, comparison, and similarity between achievement competencies, in the sense of those that are not sustainable (unoccupied) and those that are steady (occupied).
- At the end of the chain of these competencies/bits of knowledge, a greater understanding of the qualities of acquired knowledge, similarities, and differences with effective sustainable competence, increases efficiency and sustainability.
- A better understanding or reinforcement of the implications and features of long-term effective competence.

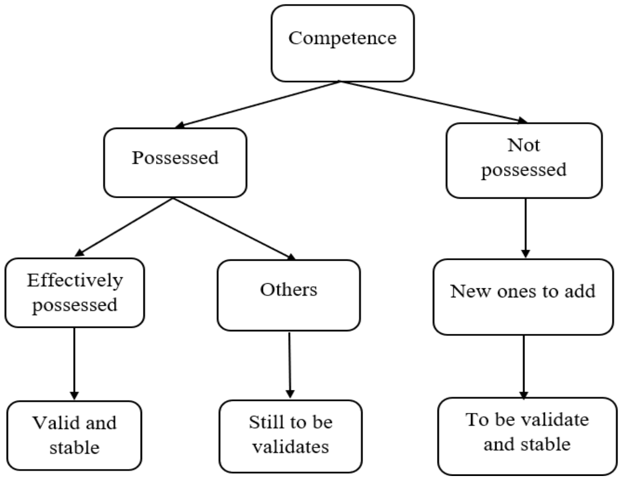


Figure 1. Scheme of competencies

The presence of errors may stimulate a need for greater accuracy in the obtained techniques or explanations, a desire that persists if the procedures or explanations generate satisfactory outcomes (Binder et al, 2018; Gagatsis & Kyriakides, 2000; Zan, 2102).

The difference between statements and counterexamples can reveal shortcomings in well-established notions and provide a specific strategy for improving them in the product-process direction (Große & Renkl, 2007; McLaren et al, 2015; Oktaviani, 2017).

Some errors can lead to new fields of inquiry and opportunities that were previously unconsidered. The student can play the part of the researcher (Borasi, 1994).

Different objects cause the same error in different subjects; hence the source of the error is crucial (Pepkolaj, 2015).

Students make mistakes without recognizing them, and if they are not recognized or studied "in the sunlight," as Montessori phrased it, it leads to more mistakes and a loss of knowledge (Montessori, 2001).

Some errors can lead to better problem-solving techniques (Pongsakdi et al, 2020).

Students that struggle with mathematics can be identified by their mistakes, which can then be used to determine the best course of action, or a method of developing understanding (Borasi, 1994; Pesci, 2012; Rushton, 2018).

Mistakes can become a source of self-correction and self-learning (Lenz et al, 2022; Lucangeli et al, 2019; Mollo, 1986; Montessori, 2001; Zan, 2012).

The analysis of the subjects' cognitive dysfunction, and their irregular periodic answers, helps us improve the processes of the functioning of didactic systems (or non-functioning), while also understanding the probable direction of treatments and most importantly, their recovery (Binder et al, 2018)

We can classify errors by studying them, which is an important parameter in structuring assistance and recovery efforts (Movshovitz et al, 1987; Radatz, 1979; Watson, 1980;)

Levels of metacognition and non-cognition

Interpretation of the error is a delicate and difficult task. It cannot be cured with a diagnostic of a faulty thought problem, therefore it's not always a gaffe with ambiguous, unexpected facts, but it's worth mentioning (Ashcraft, 2016; Baten et al, 2017; Gagatsis & Kyriakides, 2000; Keith & Frese, 2005; Núñez et al, 1999):

- Stimulating understanding of the causes of mistakes rather than just fixing them.
- Trying to understand the student's perspective rather than the experts.
- Individual or group control of errors (lecturer-student, student-student), followed by the need for the student to identify boundaries as a necessary condition to rectify them.

Only seeing the error on a cognitive level limits the ability to investigate the condition that caused it and can also obstruct any potential recovery intervention. In metacognitive and non-cognitive regions, cognitive interventions such as repetition of learning, supply of exercise materials, and problem-solving in a different method are insufficient (Hill et al, 2008; Julie & Mancy, 2018; Keith & Frese, 2005; Putwain & Wood, 2022; Zan, 2012).

The significance of error recognition and self-control

Given the necessity to recall paths and deadlines, one aspect of daily life in the past of the control process was a memory. Even in mathematics, consciousness plays an important role in activating control systems. The ability to cope with the achievement of a task assigned to them is to recognize troublesome situations by being aware of a person's talents (Lucangeli et al, 2019; Keith & Frese, 2005; Pampaka et al, 2011). Zan (2012) states, "At the same level as topic knowledge, being aware helps us to activate the right control procedures and considerably improve the output" (Zan, 2012 p. 123).

If a student is unable to manage the products, s/he will be unable to recognize the problems and may develop a sense of incompetence and discouragement, possibly leading to contempt for the topic, as is common in mathematics. Students struggle to deal with difficult situations that necessitate the use of personal competencies. When you become aware of a mistake, you can check or remedy it (Brodie, 2014; Gagatsis Kyriakides, 2000; Grassinger & Dresel, 2017). Montessori (2001) says that "One of the greatest triumphs of psychic independence, is to know that we can make a mistake and recognize and control it without help." (p. 130).

Error checking acts as a guideline to ensure we are on the right route. More than having freedom, developing critical thinking toward a particular direction, and having the ability to ask ourselves when we make a mistake, the ability to continue entails a variety of factors. Students should be able to self-regulate their faults throughout school practices. The teacher should use the constant feedback he receives from his students to give them powerful control tools (Brodie, 2014; Lucangeli et al, 2019; Pankow et al, 2018; Schneider & Artelt, 2010).

It's also important for the student to be ready to accept that he or she will make mistakes. Montessori (2001) emphasized that "Brotherhood is born better in the route of error than in the way of perfection," (p.131), and a calm atmosphere should be created. We should have error control and dealing with it would be more pleasurable than solving the problem (Große & Renkl, 2007; Semana & Santos, 2018; Tulis et al, 2017). Finally, we may conclude that Montessori's main notion is that we should strive toward an education that places children in practical settings and allows them to self-correct (Montessori, 2001).

Error checking makes way to (Große & Renkl, 2007; Keith & Frese, 2005; Semana & Santos, 2018; Schneider & Artelt, 2010; Smit et al, 2022; Zan, 2012)

- Reliability in the subject’s opinion.
- Competence and self-control validity.
- Learning.

Stimulation through the right application of errors

A greater understanding of errors in educational practices and how to remedy them considerably enhance kids' learning circumstances and outcomes. It will lessen anxieties, strengthen internal drives, increase self-efficacy, and improve the quality of success and failure for students (Ahuja, 2018; Ashcraft, 2016; Große & Renkl, 2007; Hettinger et al, 2022; Tulis et al, 2017). Correct use of errors promises (Ingram et al, 2015; Keith & Frese, 2005; Lucangeli et al, 2019; Pongsakdi et al, 2020; Zan, 2012):

- Suggestions that place trust in the study of the problem rather than the memorization of mechanical principles. This type of goal setting is like exercises that do not require any explanation and can be learned quickly. One is constantly confused as to why that example is correct.
- The troublesome situation's self-explanation or self-control.
- It serves as an incentive for a deeper observation and investigation in several ways because the result or the problematic situation does not perfectly match the expectations.
- A stimulus to explore learning after the student has analyzed the power of problem-solving, reasoning for choices, understanding the structure and plan utilized in finding the answer to the problem, finding the dependency, and the substance of the mistakes made. At the same time, the example provides a method for the student to analyze, which is distinct from the problem-solving purpose.
- A student focus that shifts from an outcome orientation to a learning orientation.

Errors must be used correctly as part of the metacognitive approach (Semana & Santos, 2018; Schneider & Artelt, 2010; Smit et al, 2022):

- In cases where mistakes are made by the same student, error identification can help develop self-monitoring skills for problematic situations.
- However, error saving should be avoidable as opposed to correct steps. Error deletion should come in a timely manner.
- Careful study of other children's mistakes within the same inaccurate example might lead to learning advantages.
- The learner can be taught to think in the other direction and, as a result, to identify interventions or regulations that unfairly cause mistakes.

Non-cognitive elements, especially learning anxiety, appear to have a significant influence in comparing errors that negatively affect their overall studies. These elements shift, particularly when the learner makes an error and considers the faults of others. As a result, studying others' incorrect examples or different approaches to tackling the same problem may be less critical emotionally than analysing one's own answers. For these and other reasons, appropriate use of erroneous examples necessitates adaptation to the student's needs, skills, and situation, ensuring that he can detect and correct mistakes (depending on the skills he possesses, depth, and previously viewed tasks) and that the student's image of himself is not harmed (Ashcraft, 2016; Hettinger et al, 2022; Keith & Frese, 2005; Núñez et.al, 1999; Putwain & Wood, 2022; Zan, 2012).

Dealing with errors in Albanian schools

In Albania, the error itself is considered also as a didactic source, and it is still used in this way. This is noted in the topics of the professional networks meetings and school curricula teams. Teachers are ready to offer assistance and guidelines in order to make students understand the mistake and learn from it. This is due to domination of a behavioural traditional approach that causes a rigid attitude toward errors and even influences the environment which is not really encouraging.

Likewise, due to the behavioural approaches, it is noted that often teachers deal with errors not in a fair way, i.e. they do not accept their mistakes and even stigmatise them.There are several techniques that teachers use to deal with errors in order to help them learn from their errors. Nevertheless, this does not happen with the entire spectrum of teachers and at the most suitable manner. Following are some of the most common techniques:

Constructive reaction of the teachers and for the teacher: The return process to the topic in the class is very important and help student understand what error have they done and how can they improve it in the future. The reaction must be two-sided, where the teacher and student could recognise and accept errors and focus mainly on specific mistakes and suggestions to improve them. This happens somehow, but not enough is done from the constructive and critical aspect and its improvement especially in the future, but only as an instant solution and for reasons deriving from the respective assignment where the error occurred.

Reflection: Another way to deal with errors is through reflection. This includes open-ends questions that encourage student to select on their mistakes and what they can change afterwards. Reflection can also encourage students to understand what they learnt from their errors and how to use them for improvement of their skills. This consists also as an important process in development of critical skills among students and self-correction abilities. Normally, this reflection and such open questions that encourage this reflection are used under the average. It is noted that this kind of questions and this form of treating mistakes is used more by experienced teachers.

Use of technology: Technology is used to help student deal and learn from their mistakes. It offers an immediate reaction and opportunities to practice the same mistake many times in order to give students more time to improve themselves. This is not used much due to lack of infrastructure but also because of teachers training.

Game-based activities: These activities help students deal with errors and learn from them in an entertaining and comprehensive way. Likewise, they include challenges and various levels that encourage students to experiment and try new things to develop their skills. These are used broadly or in average among young teachers.

Organisation of practical cases: These cases include situations from real life that students can face in their daily life and assistance to help them find solutions for their correction. This is organised by the teacher without focusing on the error and what caused them, but only highlighting and correcting it immediately. It is noted that only experienced teachers make students reflect on their mistakes through open questions.

Organization of teaching based on errors and their analysis, which is achieved by use of analysis instruments and elaboration of tests. The teacher notices a common error among students, which occurs on a specific exercise of the test, identified by the instrument of test analysis, where the student has realised or not the error and therefore organises a special lesson to help students deal with this error. The processing and interpretation tables of the tests results and report analyses of assignments based on tests instruments, as well as other conclusions deriving from them are used as an excellent source of data to organise teaching based on mistakes. Likewise, the problem is that drafting of tables and reports are not created properly and sometimes they do not represent all the assignments in a test. Each quarter, teachers draft these tables, but their data are not used much to really help in organization of teaching based on errors.

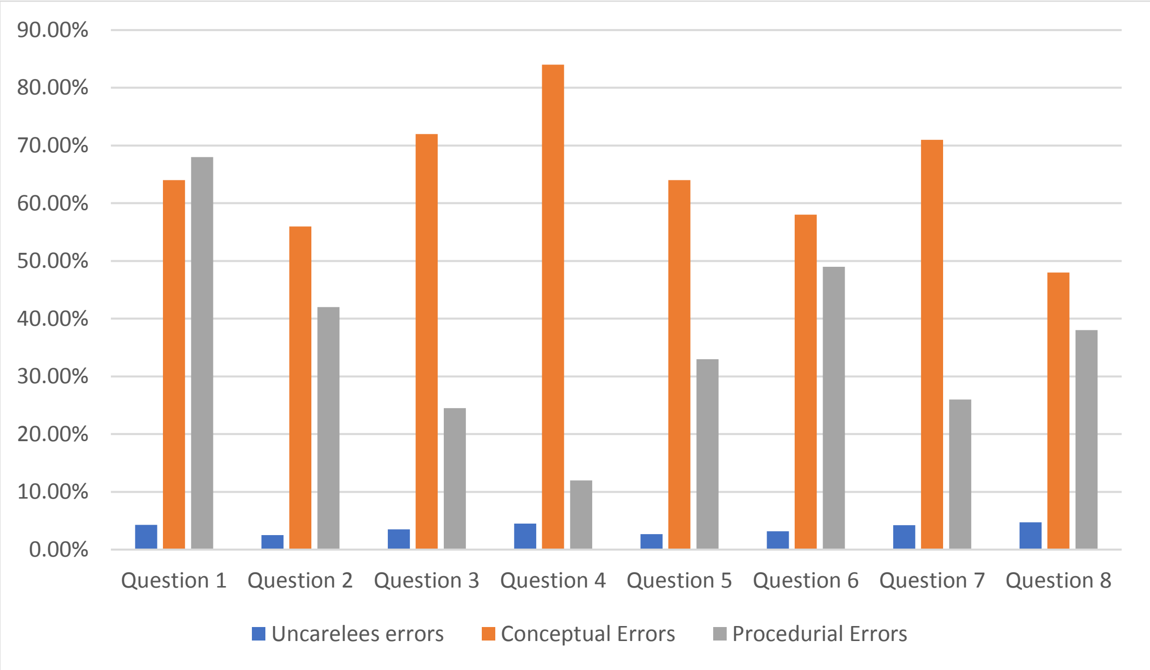
In conclusion, we should underline the following issues:

- There exist tasks or assignment in the curricula tests which contain the error and students must identify and correct it, but their role is not enough.
- Albanian schools do not embrace very well the pedagogical and psychological approaches toward errors and there is a lack of literature on them in Albanian language.
- There is little effort to work with methodologies that consider error as a didactic source. There exists mainly a behaviourist point of view regarding error, also as a result of an insufficient training, poor consideration of this problem in university curricula, school curricula experts and by professional networks.
- There is no concrete didactic plan concluded from a scientific research

A case stady in Albanian schools

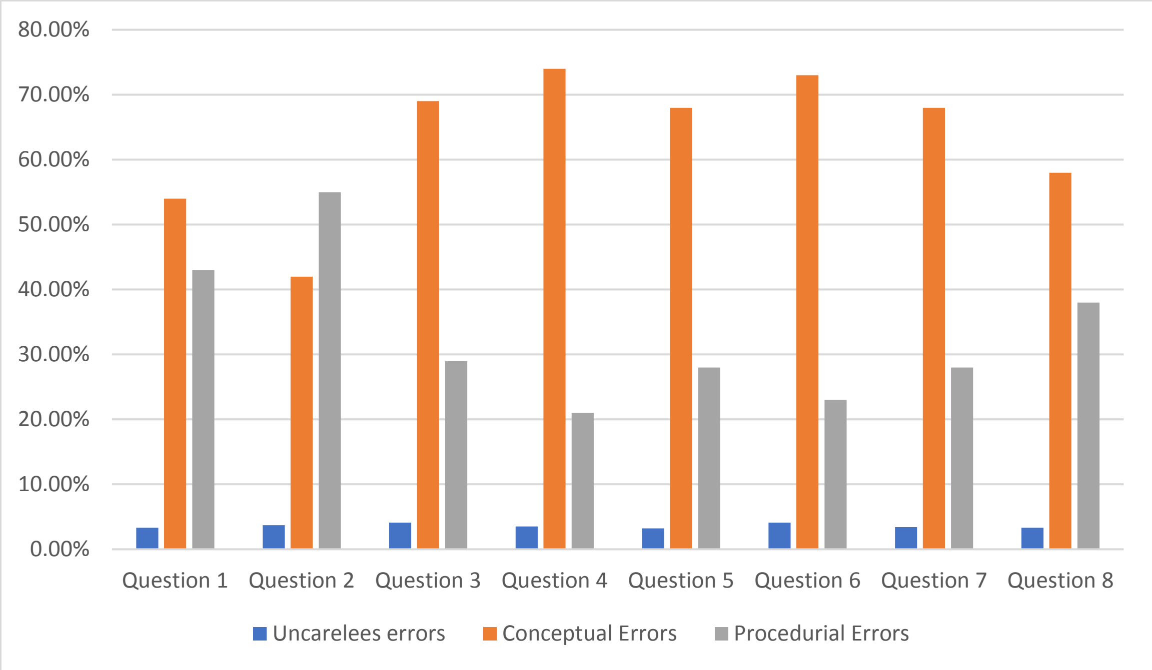
Based on a test[1], in the disciplines Analysis, Algebra, Geometry, Probability and Statistics with 192 matura students and 98 students of the first year of Bachelor in Mathematics, Computer Science and Economics, there were a few errors in solving exercises. These errors are classified by using a method called *Luneta 2011* which separate them into: careless errors, conceptual errors and procedural errors.

In the discipline of Algebra, the results are given in the following graf:

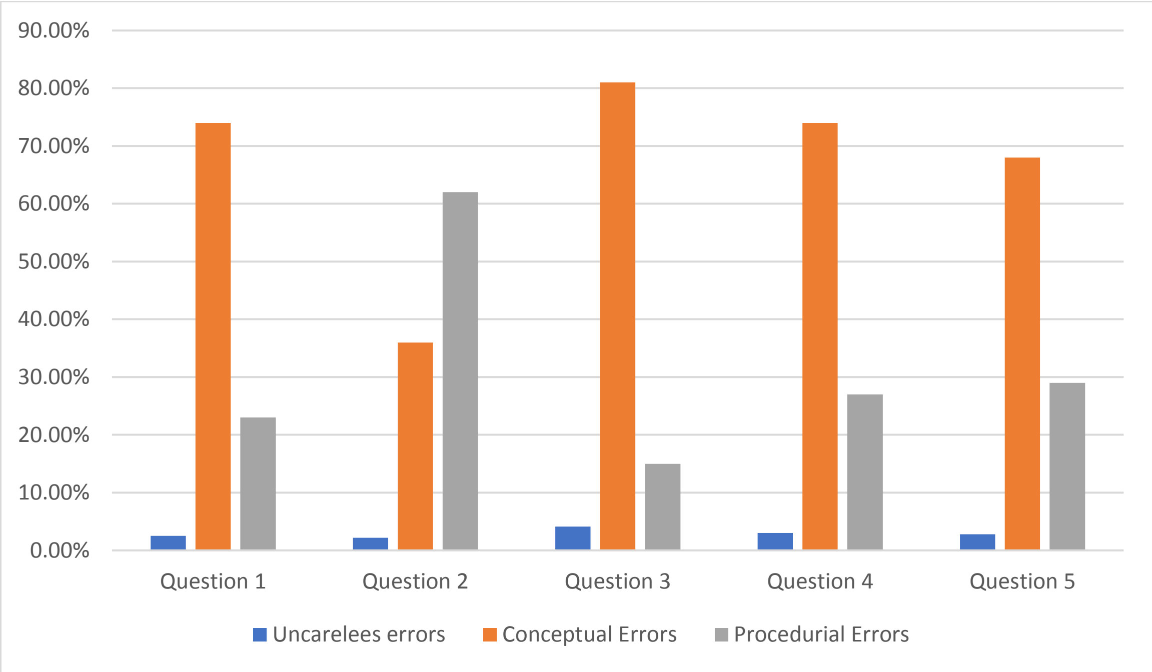


[1] *Research Project: The challenge of the new Mathematics curriculum in pre-university education and its impact on the university education system (2022).*

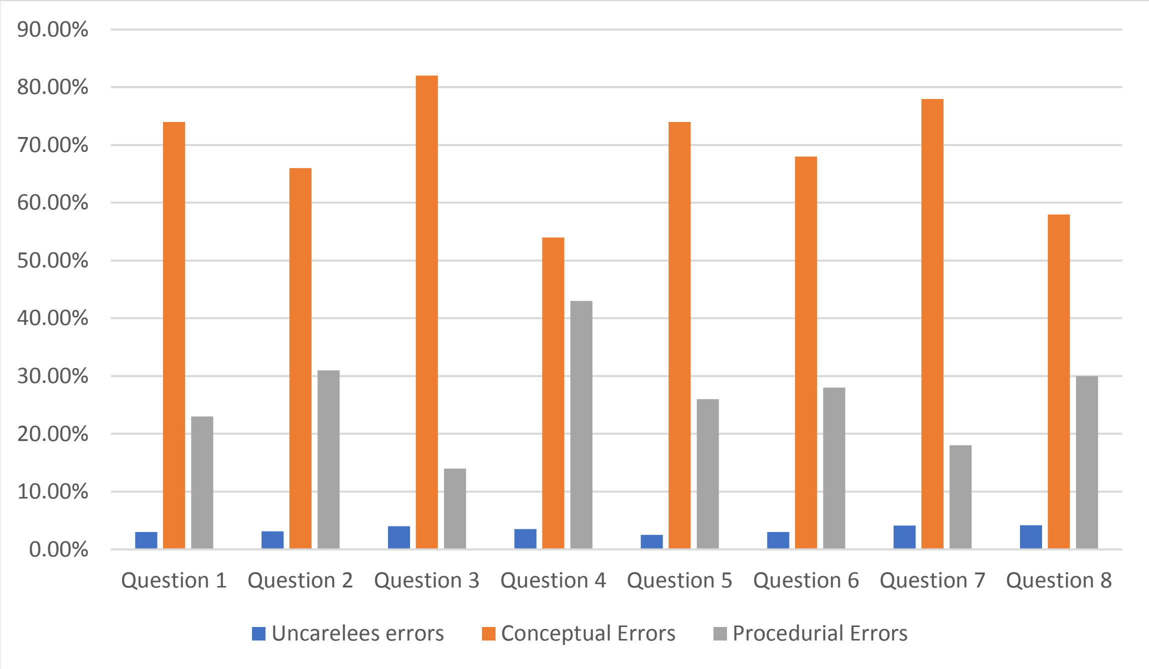
In the discipline of Analysis, the results are given in the diagram below:



The results in the discipline of Geometry, are reflected in the following graf:



The results in the disciplines of Probability and Statistics, are given in the following figure:



Conclusion

Constructivist learning theories are directly linked to the interpretation and analysis of errors. Error is a significant criterion for monitoring teaching and learning processes, as well as a "difficulty" that instructors may use to examine a subject's effective competence and how it is formed. Analysis of Errors:

- It improves effective and long-term skills.
- It acts in the product process and learning-from-results directions.
- It is a crucial factor in rehabilitation and support treatments.

The examination of the subjects' cognitive issues helps us improve the processes for the didactic system's working or non-functioning. The effective use of errors provides motivation for:

- Observing and analyzing more extensively and in more ways because the outcome or difficult circumstance do not match what one anticipates.

- Finding the substance of the mistakes made, discovering lessons, rationale for chosen pathways, and discovering the scheme used in finding the solution to the problem.
- Indeed, the ability to improve depends first of all and mainly on the awareness and self-control of competence.
- To make use of the advantages of mistakes, one must progress toward an education that takes the following two aspects:
- The subject's behavior during the reflection and observation of his or her learning.
- Their efforts, exhaustion, uncertainties, and failures may all be overcome with patience and assistance.

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