

COMPARING THE SOLOW MODEL AND THE RAMSEY-CASS-KOOPMANS MODEL FOR ALBANIAN ECONOMIC GROWTH

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Abstract

This paper seeks to compare the Solow's model and the Ramsey-Cass-Koopmans (RCK) Model to provide a forecast for the growth pattern of the Albanian economy. Success factors such as the factors behind the economic dynamism, the models focus on, the long run growth and the context of technological development and accumulation of capital are their focus. The Solow growth model regards technological advancement and the accumulation of capital as the crucial motorizes of growth. But also points out the law of decreasing returns to capital. On the other hand, the RCK Model enhances the analysis of economic growth explaining the household's behavior with respect to consumption and saving specifically to respond to the need of time preference. Therefore, it is crucial in this model to explain things around smoothing of consumption, appropriate levels of saving, the cost of the economic policy, among other things. In the context of Albania's economy, the models can be applied by using macroeconomic indicators such as growth rates of gross domestic product (GDP), work force population, amount of capital accumulated and other technological measures. The Solow and RCK mode implementation of the RCK model to the US economy has not been a straight forward procedure.

Key words: Ordinary differencial equations(ODE), Ramsey-Cass-Koopmans, solow growth model, economic growth, investment, FDI, Albania, MATLAB, python.

Përmbledhje

Ky punim ka për qëllim krahasimin e dy modelve të rritjes ekonomike, Solow dhe Ramsey-Cass-Koopman(RCK), me synim ndërtimin e një parashikimi sa më të besueshëm mbi zhvillimin ekonomik të Shqipërisë. Të dy modelet synojnë të shpjegojnë procesin e rritjes ekonomike duke përdorur modelime të ndryshme matematikore bazuar në ekuacione diferenciale. Modeli Solow bazohet në parimin e kthimeve në rënie, që do të thotë se çdo njësi shtesë e kapitalit prodhon një rritje më të vogël në prodhimin total, nëse faktorët e tjerë mbeten konstantë. Nga ana tjetër, modeli RCK e zgjeron analizën duke përfshirë sjelljen mikronivel të familjeve në lidhje me konsumin dhe kursimin, duke marrë parasysh preferencat kohore të individëve. Kjo qasje ndihmon në kuptimin më të thelluar të vendimeve ekonomike që lidhen me zbutjen e konsumit në kohë, përcaktimin optimal të kursimeve dhe kostot që burojnë nga politikat ekonomike. Në kontekstin e ekonomisë shqiptare, këto modele mund të aplikohen përmes përdorimit të treguesve makroekonomikë si norma e rritjes së Produktit të Brendshëm Bruto (PBB), popullsia aktive në tregun e punës, niveli i kapitalit të akumuluar dhe indikatorë të tjerë që matin përparimin teknologjik. Megjithatë, zbatimi i modelit RCK, siç ka ndodhur edhe në rastin e ekonomisë së Shteteve të Bashkuara, nuk paraqet gjithmonë një proces të drejtpërdrejtë.

Fjalë kyçe: Ekuacione diferenciale të thjeshta (ODE), Ramsey-Cass-Koopmans, modeli i rritjes së Solow-it, rritja ekonomike, investime, IHD (Investime të Huaja Direkte), Shqipëri, MATLAB, python.

Introduction

Since the fall of communism in the early 1990s, Albania has passed through a significant economic transformation. The transition from a centrally planned to a market-oriented economy has brought about dynamic changes. This change was accompanied by challenges and opportunities for sustainable economic growth. Understanding the drivers of economic growth is essential for policymakers. Two models that are used to analyze economic growth with different perspectives, are Solow and Ramsey Cass Koopman (RCK).

The first one Solow model, which is developed by Robert Solow in the 1950s, is focus on capital accumulation, labor, and technological progress. The second one, RCK model, developed by Cass and Koopmans in the 1960s, is based on the Solow model but it includes interporal consumption and saving

decisions by households. This model incorporates dynamic equations supervising capital accumulation and consumption. This paper aims to compare these two models and analyze their applicability to Albania's economic development.

As we are in a dynamic system, because the economic situation changes over time, those two models are very useful because they are based on differential equations. These models are easily implementable in a different programming language, but we have chosen to work in Python and MATLAB programming language.

Methods and materials

Solow model is one of the most fundamental concepts in macroeconomics. It is used to understand the denominators of economic growth over time. It is developed by two economists, Robert Solow and Trevor Swan, in the mid-20th century. According to Robert Solow contribution in this model, he was awarded the Nobel Price in Economic Science in 1987. By adding in the Rober Solow model the savings and investment decisions, Trevor Swan extended the Solow's work. Trevor Swan model aims to understand of how capital accumulation affects economic growth. The Solow growth model is used in the field of economy in order to:

- 1-Understand technological advances and economic growth, because by using this model economists can understand how each variable affects the others.
- 2-Represent an economical growth potential
- 3-Analyze the role that labor, technology and capital play in the growth of an economy
- 4-Explain market competition

Ramsey Cass Koopman model was introduced by Frank Ramsey, who in 1928 published a seminal paper "A Mathematical Theory of Saving". In his work, he was focused on the intertemporal decision-making and the implications of consumption and savings decisions over time. His work was extended by David Cass, an American economist who focused his work in formalizing the dynamic optimization problem of capital accumulation and consumption smoothing, and by Koopmans, a Dutch-American economist who highlighted the role that economic equilibrium has in understanding long term economic growth. Economic equilibrium is a steady state where main economic variable, such as capital accumulation, consumption etc., are stabilize over time.

The Solow growth model is also a dynamic system, which means that it can be changed with different production functions, depending on essential assumptions. It has four key components which are:

1-Output Production function: We can use as output production function: Cobb-Douglass or CES. In this work we are focused on Cobb-Douglass function. The output production function is used to see the role of capital and labor in the production of goods and services. Cobb-Douglas production is defined as:

$$Y = AK^{\alpha}L^{1-\alpha}$$

where:

Y is the output, in our case, the GDP

K is the capital stock

A is total factor productivity (catching up with technological progress)

L is the labor force

α is the output elasticity of capital, with $0 < \alpha < 1$.

Compared to the Cobb-Douglas production function, the assumptions about the interaction of capital and labor in production are fewer, but the constant elasticity of substitution is difficult to calculate due to the high number of parameters

2-Capital accumulation: It is assumed that savings from income are invested in new capital. Also, the rate of capital accumulation is a function of depreciation and savings. If we define the saving rate with s and the depreciation rate with δ we have that the rate of capital accumulation $\hat{K} = sY - \delta K$.

3-Population growth: Another assumption that we make in this model is that the labor force grows at a constant rate (n). Based on this assumption we can say that $\hat{L} = nL$.

4-Technology progress: If we define the rate of technological progress with g , we can find the growth rate of total factor productivity as: $\hat{A} = gA$.

These three equations(ii, iii & iv) form a system of ordinary differential equations(ODEs) that describe the evolution of labor force and capital stock over time.[Ziyi Lin , Hu Wang, 2021] The system that we need to solve is:

$$\begin{cases} \frac{dK_t}{dt} = sY_t - \delta K_t \\ \frac{dL_t}{dt} = nL_t \\ \frac{dA_t}{dt} = gA_t \end{cases}$$

remembering that : $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$.

In this work, in order to analyze the impact that FDI has in the economic growth in Albania and the role of capital accumulation, population growth, and technological progress, we use the above Solow growth model.

By implementing the FDI the system of differential will be change. The new system will be:

$$\begin{cases} \frac{dK_t}{dt} = (1 - \delta)K_t + sY_t + FDI \\ \frac{dL_t}{dt} = nL_t \\ \frac{dA_t}{dt} = gA_t^\alpha \end{cases}$$

RCK model aims to analyze intertemporal decision-making and capital accumulation. To achieve this goal, this model involves dynamic optimization techniques and differential equations. So, in this model the key mathematical concepts are:

➤ **Dynamic optimization**

Here we have to formulate an optimization problem where the objective function represents the agent's utility, and the set of the constraints represents the resource and technology constraints. By optimizing the objective function subject to these constraints, interested parts can derive the optimal path of consumption and investment over time. In a few words, here we aim to solve the intertemporal utility maximization problem inherent in the RCK model. Mathematically it is represented as:

$$\max_{C(t), K(t)} \int_0^{\infty} e^{-\rho t} (U(C(t))) dt$$

where:

- $C(t)$ is the consumption at time t
- $K(t)$ is capital at time t
- $U(C(t))$ is the utility function representing the agent's preferences
- p is the discount rate. A high value reflects high impatience.

This problem is known as social planner, which is a constrained optimization problem which attempts to maximize some notion of social welfare. Social welfare is a function that ranks a set of social states by their desirability.

➤ Differential equations and production function

A production function helps to represent the relationship between inputs and output in an economy. The inputs are for example capital, labor etc. and the output are good or services. In a few words, the production function describes how much output can be produced from various combinations of input. This function in its general form is represented as:

$$Y = F(K, L)$$

where:

- Y is the level of output produced
- K is the quantity of capital input
- L is the labor force
- $F(\cdot)$ is the production function.

There are some types of production functions such as:

- ❖ Cobb-Douglas Production Function
- ❖ CES Production Function
- ❖ Translog Production Function

Between those three functions, the Cobb-Douglas Production Function, is the most widely used because of its simplicity and tractability. This function is defined as:

$$Y = AK^\alpha L^{1-\alpha}$$

where:

- A is the total factor productivity
- α is the output elasticity of capital

After defining what kind of production function, we will use now we can write the capital accumulation equation and the consumption dynamics. The

capital accumulation equation is used to calculate the rate of change of the capital stock(K) over time (t). It is defined as follows:

$$\dot{K}(t) = \frac{dK}{dt} = sY(t) - \delta K(t)$$

where:

- $\dot{K}(t)$ is the rate of change of capital stock
- s is the savings rate
- $Y(t)$ is the output at time t .
- δ is the depreciation rate of capital

The in RCK model we have the following equation:

$$\frac{dC(t)}{dt} = \frac{dU}{dC} \cdot (r(t) - p)$$

that describes the rate of change of consumption (C) over time (t). In this equation:

- $\frac{dC(t)}{dt}$ is the rate of change of consumption
- $\frac{dU}{dC}$ is the marginal utility of consumption
- $r(t)$ is the interest rate at time t
- p is the rate of time preference.

Results and discussions

To implement the Solow Growth model we have taken the following parameter:

- saving rate
- depreciation rate
- technology growth
- initial capital
- FDI inflow

These parameter are taken from the Albanian Institute of Statistics, the Bank of Albania and other public sources[INSTAT,Data World Bank]. We have done two approaches , in the first one we haven't take into consideration the FDI and in the second one we have consider it. Figure 1 shows that technology is advancing , but the labor force is going to decrease. We have to take into

consideration the fact that in Albania the immigration is in high levels. Also, the Albania population is not “familiar ” with the new technology, which causes the labor force to decrease with the evaluating of technology.

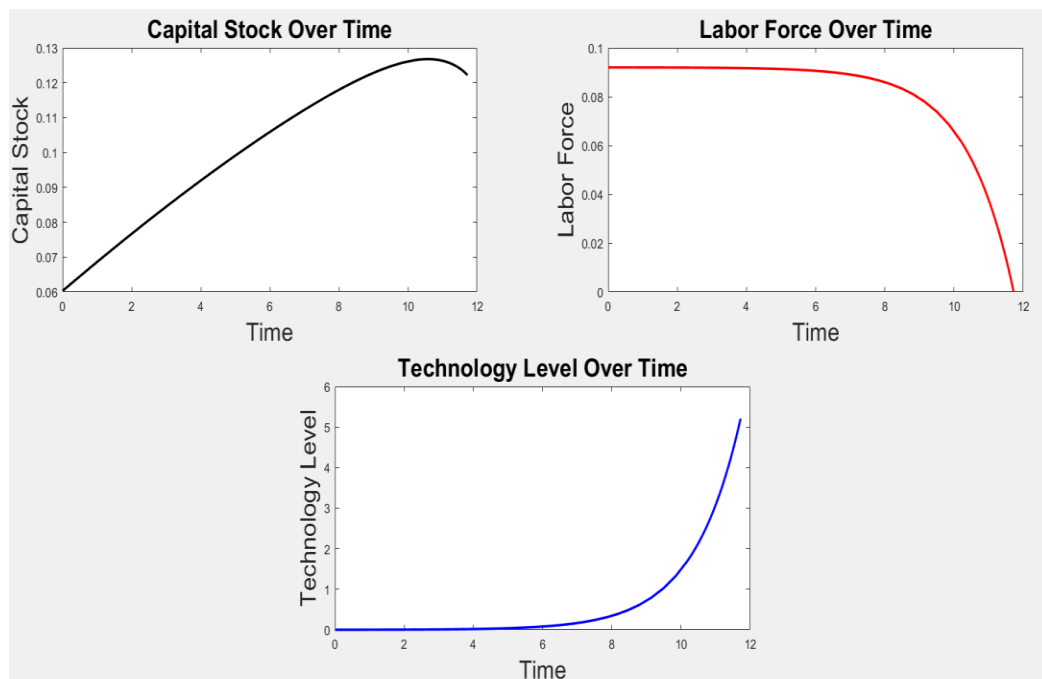


Figure 1. Solow model without adding FDI

In figure 1 the results when we implement the FDI are shown. It is seen an increasement of capital stock and the GDP. This is expected, because from 2022 to 2023 the FDI has been increased[ceicdata,].

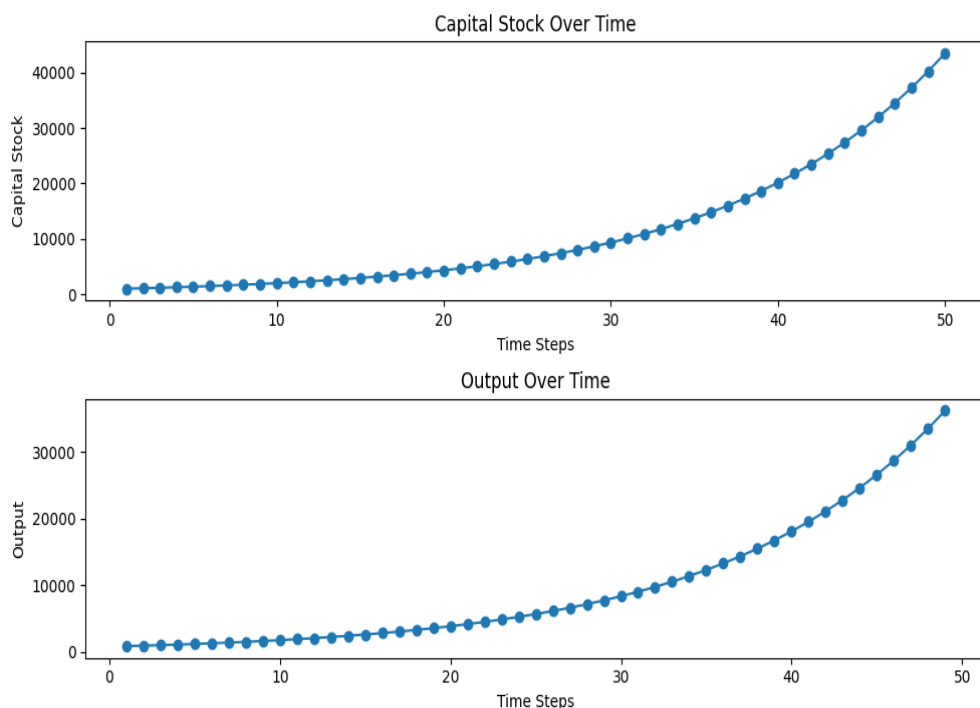


Figure 2. Solow with FDI

To implement the Ramsey Cass Koopman model to analyze the economic situation of Albania we have taken the following parameters from the Albanian Institute of Statistics, The Bank of Albania and another public sources[INSTAT, Data World Bank]:

- saving rate
- depreciation rate
- consumption
- GDP
- investment
- population growth

For consumption, GDP, investment and population growth, we have taken data from 1997 to 2022 (yearly data). In the Table 1, we can see a part of this database.

Table 1. Albanian data from 1997 to 20022

Data	Consumption	GDP	Investment	Population Growth
1997	-17.5	-10.9	2.1	-0.6
1998	-5.7	8.8	1.8	-0.6
1999	2.6	12.9	1.3	-0.6
2000	4.4	6.9	4.1	-0.6
2001	4.6	8.3	5.3	-0.9
2017	2.5	3.8	7.9	-0.1
2018	2.9	4.0	7.9	-0.2
2019	3.3	2.1	7.8	-0.4
2020	-2.8	-3.3	7.1	-0.6
2021	5.1	8.9	6.8	-0.9
2022	5.5	4.9	7.6	-1.2
2018	2.9	4.0	7.9	-0.2

From 1997 to 1999, it can be observed that GDP and Consumption have increased drastically. We know that this period has been a transitional period for Albania as we moved from a centralized economy to a free market economy. The same trend is with investments, especially they have increased after 2006.

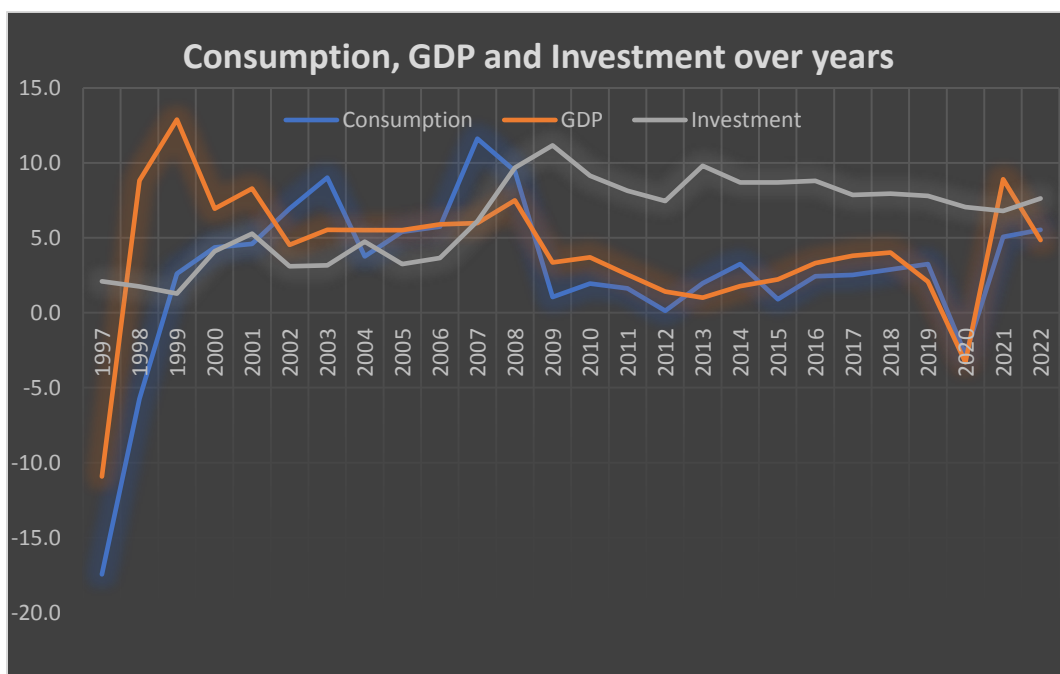


Figure 3. GDP, Consumption and Investment over the years.

By implementing the Ramsey Cass Koopman model using those data we can see that GDP, Consumption and Capital will increase over time. Especially, in GDP and Consumption we can see a rapid growth in the first 3 years.

Then the growth is smaller. While for capital, we see that we have a continuous growth. We can expect this because we said before that the investment have been increased over time. Increased investment leads to capital accumulation, fueling economic growth.

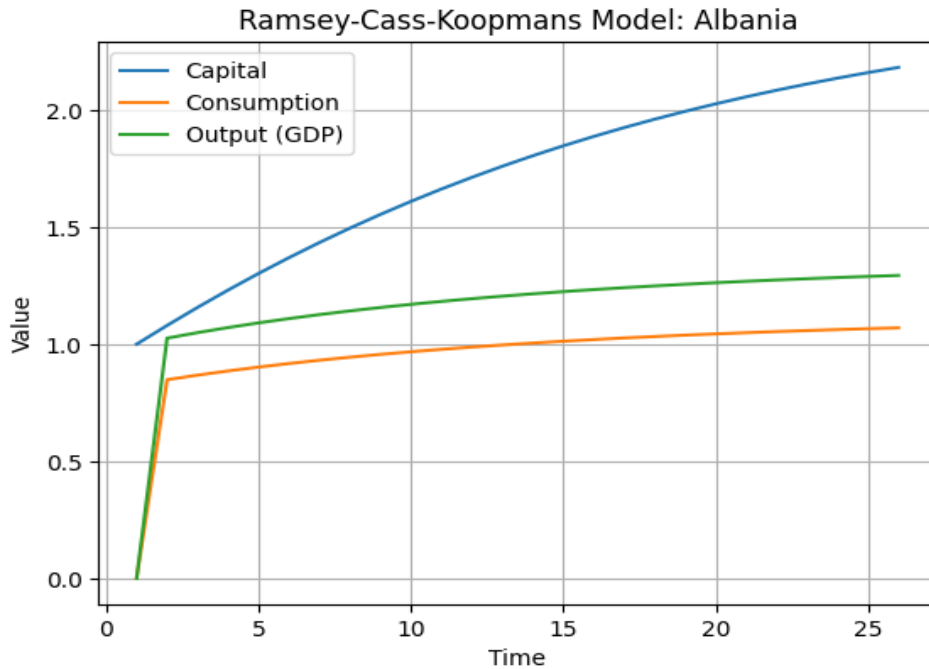


Figure 4. Ramsey-Cass- Koopman Model

Conclusion

In conclusion, in many disciplines ordinary differential equations are present. These equations are important as they serve to express and solve the dynamic relationship between several variables over time.

They can be implemented in many mathematical models and one of them is the Solow growth model. This forecast is based on several factors such as population growth, capital accumulation and technological progress.

But as we know, foreign direct investments are an important factor in predicting the economic growth of a country. In the case of Albania, we see that with an increase in technological progress, we have a decrease in the

workforce. If we add FDI to the model, we have an increase in GDP and capital stock.

On the other hand, Ramsey Cass Koopman, which is a model introduced by Frank Ramsey and extended by David Cass and Koopmans, aims to analyze intertemporal decision-making and capital accumulation. In this model we use dynamic optimization, differential equations and production function.

As production function, we can use Cobb-Douglas Production Function, CES Production Function or Translog Production Function. The Ramsey Cass Koopman model not only helps to understand historical trend but also helps policy makers to take measures to have a sustainable economic situation. This model is used to analyze the complex dynamic of economic growth, incorporating essential factors that affect the economic growth of a country.

We have used saving rate, depreciation rate, consumption, GDP, investment, population growth to see the evolution of Albania economy. We observed that GDP and consumption growth was rapid in the first three years while capital growth was steady. The rapid GDP and consumption growth underscored a period of economic expansion, while steady capital growth demonstrated sustained investment in productive assets.

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