STUDY OF BIFURCATIONS AND STABILITY IN NEURONAL OSCILLATION MODELS THROUGH NUMERICAL ANALYSIS

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Abstract

Potential oscillation in neurons is quite well-known and presents its own challenges in in-depth studies of the theoretical models proposed by different researchers. The self-contained oscillatory behaviour of the neuron's potential after an external stimulus as well as other bifurcation phenomena add to the need for the analysis of the nonlinear dynamics of the modelled system by advancing to computer simulations for different values of the stimulus magnitude, enabling a systematic investigation of the parameter space of mathematical models. Numerical integrations and computer simulations provide clear insight about the behaviour of the complicated systems, pointing out various phenomena such as changes in the stability of systems. Neuronal membrane potential oscillations represent a well-known nonlinear phenomenon where bifurcations and other nonlinearity phenomena appear for different values of the control parameter. In the following, the time series, phase space, parameter space and bifurcation diagrams will be presented for the basic mathematical models of neurons such as the Fitzhugh-Nagumo (FN) and Hodgkin-Huxley (HH) models, and other more advanced models will be attempted.

Key words: *Neuronal oscillation, nonlinear dynamics, bifurcation, numerical simulation.*

Përmbledhje

Dukuria e oshilimit të potencialit të neuroneve është mjaft e njohur dhe shfaq sfida më vete në studime të thelluara të modeleve teorike të propozuar nga *studiues të ndryshëm. Sjellja oshiluese e vetëpërmbajtur e potencialit të neuronit pas një ngacmimi të jashtëm si dhe dukuritë e tjera të bifurkacionit shtojnë nevojën për analizën e dinamikës jolineare të sistemit të modeluar duke avancuar në simulime kompjuterike për vlera të ndryshme të madhësisë ngacmuese, rrjedhimisht në studimin e hapësirës së parametrave të modeleve matematik. Integrimet numerike dhe simulimet kompjuterike na japin informacione të qarta të sjelljes së sistemeve të komplikuara duke na vënë në dukje dukuri të ndryshme siç janë ndryshimet e stabilitetit të sistemeve. Oshilimi i potencialit të neuroneve është një rast konkret ku bifurkacione dhe dukuri të tjera të jolinearitetit shfaqen për vlera të ndryshme të parametrit të kontrollit. Në vazhdim do paraqiten seritë kohore, hapësira fazore, hapësira e parametrave si dhe diagramat e bifurkacionit për modelet bazë matematik të neuroneve siç janë modeli Fitzhug-Nagumo (FN) dhe Hodgkin-Huxley (HH) dhe do tentohet në modele të tjera më të avancuara.*

Fjalë kyçe: *Oshilimet e neuroneve, dinamika jolineare, bifurkacion, simulim numerike.*

Introduction

Neurons have evolved elaborate mechanisms for generating electrical signals based on the flow of ions across membranes. The diffusion that occurs in the membrane is characterized by the Nernst-Planck equation (A. L. Hodgkin & A. F. Huxley, 1951). The phospholipid double layer acts as a capacitor with parallel plates, separating the charges (inside and outside the cell), therefore electric circuit is used to create the theoretical model (F. Giovannini, 2017; Joan Bisquert, 2021).

In this study, we will focus on the oscillation of electrical signals in a single neuron when excited by external stimuli (Hodgkin, 1937) which is a phenomenon known and studied by researchers in various fields (Hodgkin & Huxley, 1945). However, the development of advanced methods for simulations and calculations of the behaviour of systems shows continuous interest in the scientific world. Several different mathematical models have been proposed to describe this oscillation phenomenon, such as Hodgkin-Huxley (HH), Morris-Lecar (ML), Fitzhugh-Nagumo (FHN), etc, (Dimitrichev, 2018; FitzHugh, 1961; Morris & Lecar, 1981; Nagumo et al.,1962) where dynamical behaviours with an oscillatory character are observed and in special cases rare phenomena such as Hopf bifurcations and chaotic oscillations appear (Wang et al., 2023).

Wang et al. has simulated and reported oscillations and 2 bifurcations occurring in the HH system. Dimitrichev et al. has done a review of all theoretical models, which emphasizes that ML, FNH and others are simplifications of the model closer to the biological case such as HH, also states that in some ideal and complicated cases strange attractors and chaotic behaviour are encountered as in the Curbage-Nekorkin model (Courbage et al., 2007; Dimitrichev, 2018). Numerous experiments and simulations (Zhong et al., 2025) have been performed to verify the oscillation mode of the neuronal potential, e.g. Hubert Eichner and Alexander Borst have created the electrical circuit according to the HH model and have evaluated the oscillations and dynamics of the system by means of an oscilloscope (Eichner & Borst, 2011). Also, machine learning techniques and neural networks have recently been developed in the application and simulation of theoretical models of neuronal oscillation (Centofanti et al., 2024).

In this study numerical integration was performed using MATLAB with the ODE45 and ODE23 packages and comparisons were made with other methods on which the ODE package is based, such as the simplest Euler method and the Runge-Kutta-4 method (Boriçi, 2020; Gega & Prenga, 2024; Gega et al., 2023). Initially, the FN model was considered, and corresponding numerical integrations were performed for different current values, and the bifurcation diagram was simulated. We continued with the HH model where the necessary simulations were performed for the case when the current is for short moments of time, and for different cases of its values. The time series, the currents of the respective channels, the opening probabilities of the channels as well as the phase spaces of the system oscillation are simulated. Also, the bifurcation diagram obtained from the phase spaces is shown.

Simplified model: Fitzhugh Nagumo

FitzHugh (1961) and, independently, Nagumo (1962) derived the following two equations to describe qualitatively the events occurring in an excited neuron (Faghih, 2010; Wallisch, 2014; Marcolli & Tsao, 2017);

$$\frac{\mathrm{dV}}{\mathrm{dt}} = V - \frac{V^3}{3} - W + I$$

(1)

$$\frac{dW}{dt} = \Phi(V - a - bW)$$

(2)

Parameters a, b, and ϕ in the FHN model are typically determined through qualitative dynamical analysis and quantitative fitting to experimental membrane potential data, rather than direct biophysical measurement. The amplitude φ, which corresponds to the inverse of a time constant, determines how fast W changes relative to V. The Jacobian is calculated and results to be $\begin{pmatrix} 1 - V^{*2} & -1 \\ \phi & -b\phi \end{pmatrix}$. From where, the linearized equation is:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 - V^{*2} & -1 \\ \phi & -b\phi \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

(3)

From calculation of eigenvalue for a=0.7; b=0.8; Φ=0.08 (Izhikevich, 2007), we can determine the stability of system for example, *I*=0: $\lambda_1 = -0.12, \lambda_2 = -0.39$, where stable fixed point characterize the system of equation. For *I*=1; $\lambda_{1,2} = 0.38 \pm 0.18i$ which indicate for unstable spiral of system. For *I*=2; $\lambda_1 = -0.08, \lambda_2 = -1.10$, the system comes to stable fixed point.

For a certain value of the current, the system undergoes a Hopf bifurcation, transitioning from a stable point to an unstable point surrounded by a stable limit cycle. If we further increase the current, the system again loses the limit cycle by jumping into another spiral towards the centre. Thus, our system exhibits two Hopf bifurcations (Figure 1 and 2) (Kznetsov, 1998).

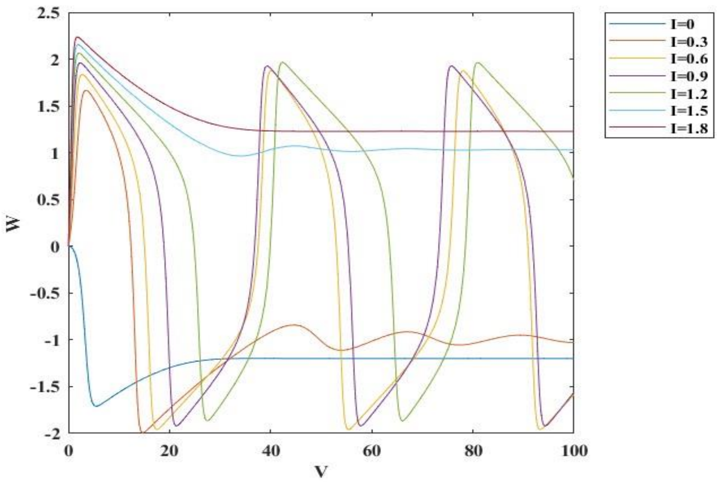

Figure 1. Potential oscillation for I=0 to I=1.8 with step 0.3 (with parameters value a=0.7; b=0.8; Φ=0.08 (Izhikevich, 2007)).

A similar model of relaxation oscillations describes the heartbeat by modelling it with an electrical circuit from which the differential equations for the relaxation oscillation are derived (Van der Pol & Van der Mark, 1928; Hafeez, Ndikilar, 2014). In Figure 1, is presented the time series of the potential oscillation for different values of the control parameter, which in this case is the current I, which varies from I=0 to I=2 with a step of 0.3 units. This range of values was taken as it was considered sufficient to capture the dynamic phenomena that interest us. In this study, the bifurcation analysis is performed with respect to the external current I, which acts as the primary control parameter of neuronal excitation. Other parameters, such as conductance's and membrane potentials, are kept fixed since they correspond to intrinsic physiological properties of the neuron and are not expected to vary significantly under normal conditions.

Therefore, varying I provides a meaningful and physically relevant way to explore the dynamical regimes of the system. The figure demonstrates that for low current values the system does not oscillate but concentrates towards a fixed point, while for some medium current values the system oscillates according to a limit cycle. Also, for high current values the system remains at a fixed point which has a higher potential value.

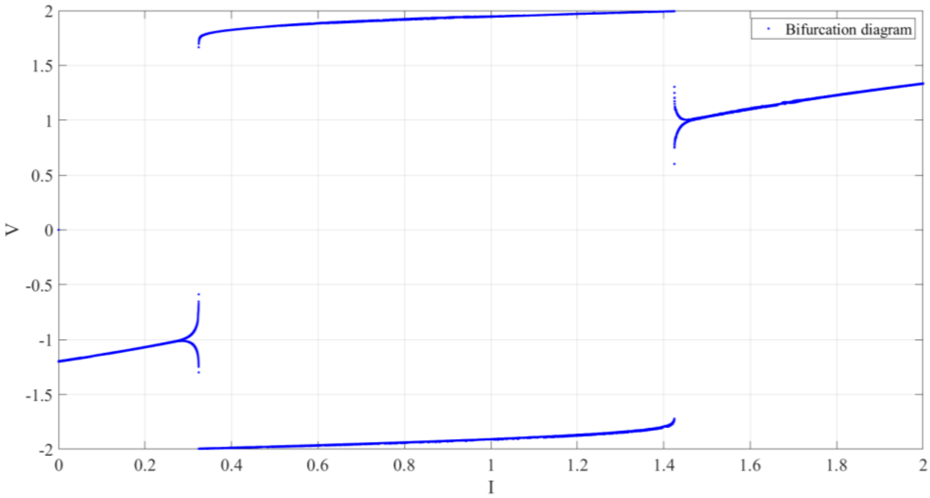

Figure 2. Bifurcation diagram of FN model (with parameters value a=0.7; b=0.8; Φ=0.08) for current value from 0 to 2 (arbitrary unit)

While in Figure 2 is presented the bifurcation diagram for current values from 0 to 2 (arbitrary unit) where the numerical integrations were performed for a time range from 0 to 300-time units with a step of 0.001 unit for each current value with a step of 0.0001. These step values have been sufficient to get a clear picture of the system. It has not been necessary to use smaller steps since the system has only some periodicity and stability in oscillation. So, in numerous simulations it is observed that the system does not have many unexpected jumps except for two bifurcations. Also, ODE 45 has been evaluated as suitable for the integration of the system combined with tests by ODE23 which overcomes the difficult moments where ODE45 may fail to converge.

For each of the current values, the numerical integration values aligned in the matrix were obtained from which the points for the construction of the bifurcation diagram were obtained. Bifurcation diagrams are formed by two methods, one consists in their construction from Poincare maps which we obtain for each value of *I* thus forming a three-dimensional bifurcation diagram, from which, by rotating I by an angle π/2 about the axis W take the normal bifurcation diagram. Since the oscillation does not exhibit chaotic behaviour, but only bifurcations, limit cycles, and fixed points, it is considered reasonable to obtain the maximum and minimum values of the numerical integrations for each value of I. We emphasize that the numerical values are selected beyond step 30 since the system passes several steps to achieve stability, limited cycles, and fixed points.

Hodgkin-Huxley model

The HH model is more realistic with respect to the oscillation of the neuronal potential compared to the experiment performed with a squid axon. Numerous studies conducted have further confirmed the assumptions made by various researchers. However, this study aims to study known models in terms of nonlinear dynamics and bifurcations of proposed systems by means of numerical methods. Deep biological analysis of the oscillation of a single neuron is beyond the context of this paper. Also, the mathematical validation of the models is considered as something known and sufficiently studied by other researchers.

The HH model presented in the following equations considers several currents that flow across the cell membrane tending to reach the equilibrium of the neuron-intercellular environment system. The currents that pass through the membrane are sodium currents (I_{Na}), potassium currents (I_k) and flowing currents, mainly chlorine (Cl) where for each of them the membrane exhibits a certain resistance or conductance which is related to the probability (mainly function sigmoid) of opening "gates" or ion channels. The mathematical model is given (Brown, 2010; Botero et al., 2012):

$$\frac{dV}{dt} = -[\bar{g}_{Na}m^3h(V - E_{Na}) - \bar{g}_K n^4(V - E_K) - g_L(V - E_L) + I]/C \qquad (4)$$

Where V is the potential, $\bar{g}_y$ is the specific conductance for each of the channels (where y ∈{Na, K, l}), and n, m, h are the opening probabilities of the channels, E_y is the initial potential of each of the channels, C is the capacitance of the capacitor modelling the capacitance appearing between the two parts of the neuron membrane. In this equation I is excitation current or external impulse of neurons which will take some different value to learn what will happen with integration of differential equations.

The gate opening probabilities are time-dependent and time-varying according to the model (Hodgkin & Huxley, 1952; Dimitrichev, 2018):

$$\frac{dx}{dt} = \alpha_x(1-x) - \beta_x x \tag{5}$$

Where $x \in \{h, n, m\}$ and the variables α_x and β_x are proposed according to the following models (Hodgkin & Huxley, 1952; Siciliano, 2012; Leander Anderson et al., 2013):

$$\alpha_m = 0.1\frac{V+35}{1-e^{-\frac{V+35}{10}}} \tag{6}$$

$$\beta_m = 4e^{-0.0556(V+60)} \tag{7}$$

$$\alpha_n = 0.01\frac{V+50}{1-e^{-\frac{V+50}{10}}} \tag{8}$$

$$\beta_n = 0.125e^{-\frac{V+60}{80}} \tag{9}$$

$$\alpha_h = 0.07e^{-0.05(V+60)} \tag{10}$$

$$\beta_h = \frac{1}{1+e^{-0.1(V+30)}} \tag{11}$$

Mathematical models of channel opening probability are different, but we have considered the above model and will attempt numerical simulations and dynamics studies for this case. The parameters considered in our simulations are $\bar{g}_l$=0.003 mS/cm^2, $\bar{g}_k$=0.36 mS/cm^2, $\bar{g}_{Na}$=1.2 mS/cm^2, E_l=-49.42 mV, E_k=-72.14 mV, E_{Na}=55.17 mV, and C=0.01 µF/cm^2 (Siciliano, 2012; Leander Anderson et al., 2013). The Jacobian and its eigenvalues have been calculated by many studies, and it has been concluded that these systems such as HH and FHN have complicated dynamic changes with oscillations and bifurcations (Wang et al., 2025).

Numerical integration of HH model

The necessary simulations were carried out considering 5 ordinary differential equations dV/dt, dn/dt, dm/dt, dh/dt and dt/dt=1 which were numerically integrated according to some known methods, by means of the method Euler, Runge Kutta-4, ODE45 and ODE23 using MATLAB and it is found that ODE45 provides higher accuracy and ease in performing the numerical integrations required for this article.

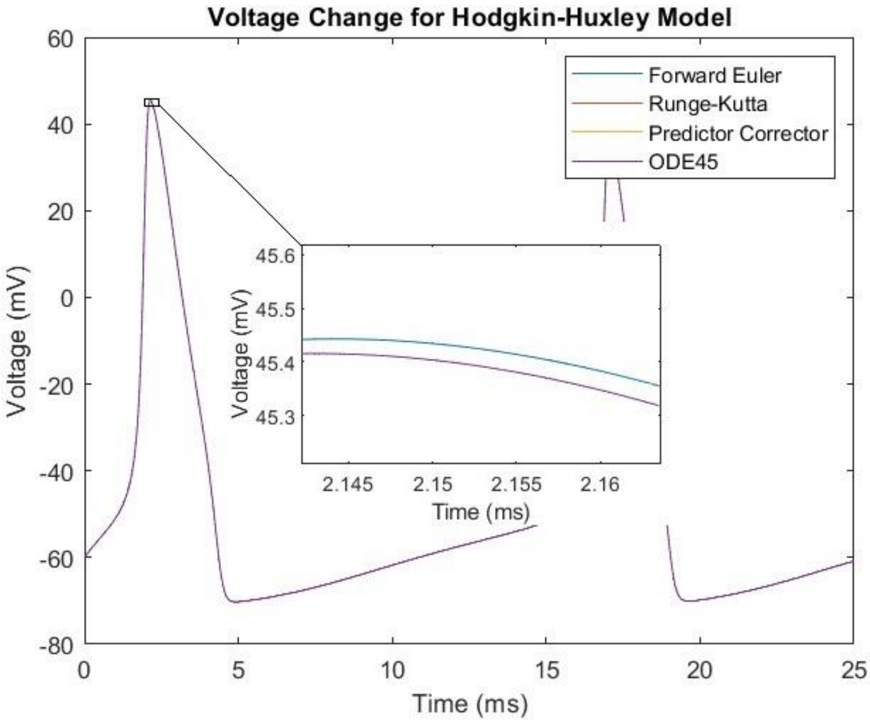

Figure 3. Comparison between several numerical integration methods. The methods used in integration are Euler, Runge Kutta 4, predictor corrector and ODE-45.

In a simple comparison (Figure 3) it results that the Euler method deviates a lot from other methods that have been described as quite accurate in numerical integration and in predicting the behaviour of systems. For the Euler, Runge Kutta, predictive corrector methods, the step dt=0.001 is used, which for the ODE45 model is set as the maximum allowed value that can be used as a possible step by the ODE.

Figure 4 shows the graphs of the behaviour of the system for a short current pulse, and the oscillation of the potential is shown, where the neuron reaches stability again shortly after the pulse. Oscillations of gate opening probabilities as well as phase space are also presented.

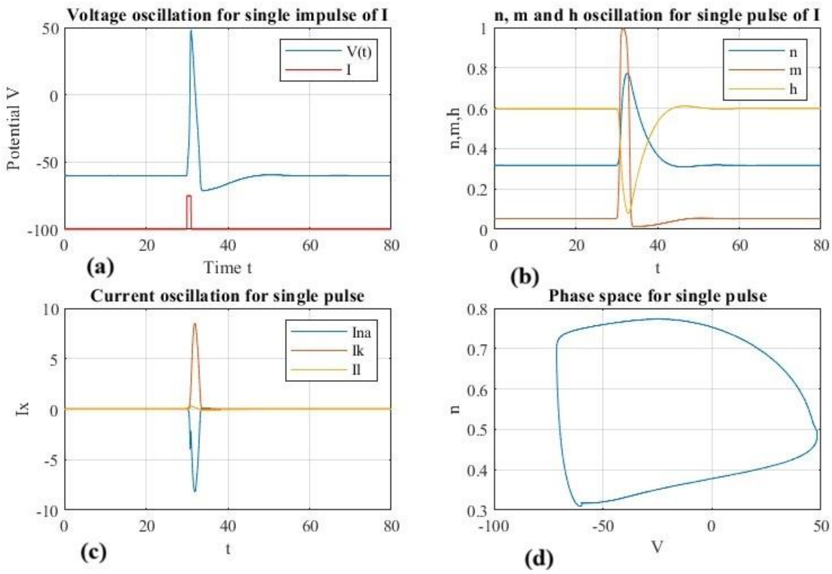

Figure 4. Numerical integration for a single pulse of the current *I*. (a) The time series of the oscillation of V, (b) of the variables n, m, h, (c) of the currents *I* and (d) the phase space.

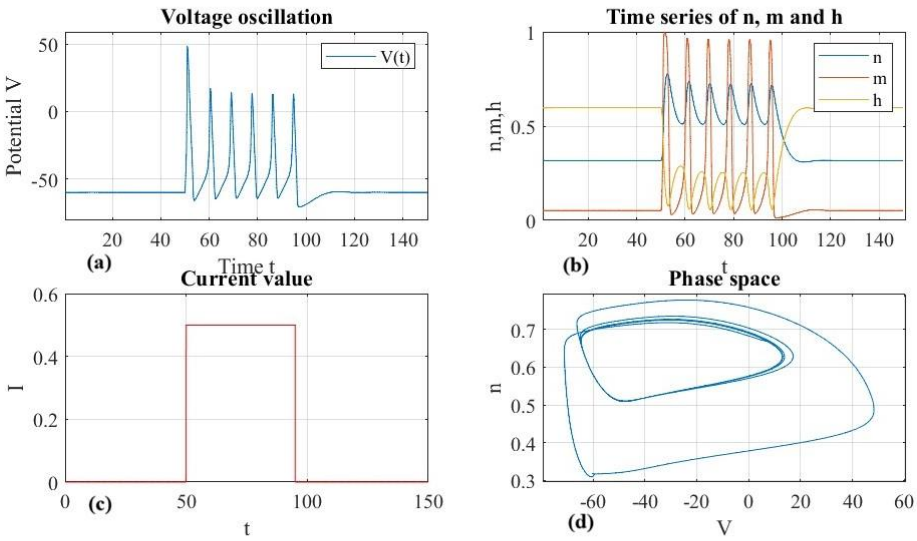

Figure 5. Potential oscillation (mV) for one fixed value of current *I*. (a) The time series of the oscillation of V, (b) of the variables n, m, h. (c) the current impulse (*I*=0.5) and (d) the phase space.

As can be demonstrated from the phase space, the system reaches the limit cycle and leaves the limit cycle again after performing a complete orbit according to it. In Figure 5, a longer period of excitation by current I, is taken into consideration, but with the same value as in the first case. As can be seen, the potential oscillates if the system is excited by the current *I*=0.5. The same happens with the opening probabilities of gates n, m and h. The system reaches the limit cycle and exits it again after the external excitation is stopped.

The following figure (Figure 6) shows the time series for different values of current *I* where we can see more clearly how they vary for several different values of *I*. But a more accurate understanding of the bifurcations can be made from the bifurcation diagram as presented in the following section.

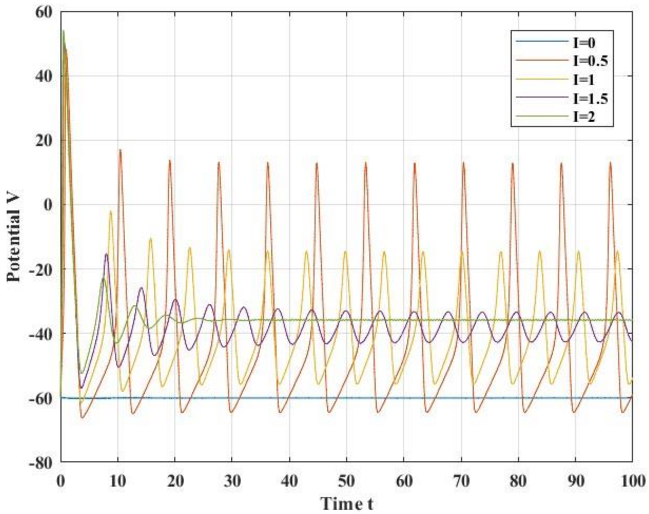

Figure 6. Time series of oscillation for some value of excitation current I (0, 0.5, 1, 1.5, 2).

Bifurcation diagram of HH model

Bifurcation diagrams give us a clear model of system stability. We have performed the numerical integration for intervals from 0 to 300 with a step of 0.01 and for current values from 0 to 2 with a step of 0.001 by selecting the iterations above the step 50. In Figure 7 is demonstrated a change in the stability of the system in V, n and I coordinate to understand in more detail what happens to the system for different values of the control parameter.

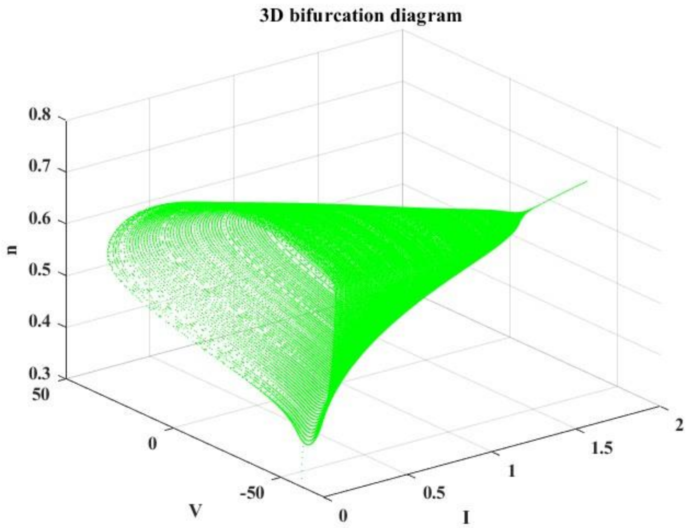

Figure 7. Three-dimensional diagram of phase spaces in function of control parameter I.

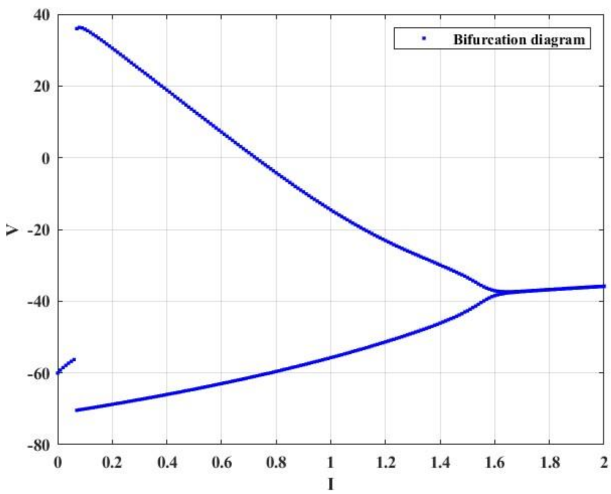

Figure 8. Bifurcation diagram of HH model

Figure 8 shows the bifurcation diagram which is simulated by numerical integration of differential equations. As seen in the figure, the system exhibits two Hopf bifurcations (Strogatz, 2015; Kznetsov, 1998), one for I=0.06 and one for I=1.63. The case of the first bifurcation represents the jump, which is consistent with a Subcritical Hopf bifurcation, as the oscillation amplitude increases smoothly from zero of the system from a fixed point to oscillation with a large amplitude. Amplitude of oscillation decrease, with increase of exited current, until the oscillation returns to full stability at another fixed point (reverse supercritical Hopf Bifurcation) but already for a higher value of the potential oscillation, about -40mV and increasing with the further increase of current I. Using Lyapunov exponents, we have proven that the system does not have chaotic effects for the above-mentioned parameters. This has been achieved by simulating trajectories for different initial values and close to each other, where it has resulted that these trajectories converge to fixed points and limit cycles.

Discussion

The comparative analysis of the FHN and HH models highlights fundamental differences in their ability to represent neuronal dynamics, particularly in terms of physiological realism, mathematical complexity, and bifurcation structure. The FHN model serves as a reduced and qualitative approximation of neuronal excitability, capturing the essential features of action potential generation through a two-dimensional dynamical system. Its simplicity allows for clear geometric and analytical interpretation in phase space. In this framework, bifurcation analysis is more transparent, and the emergence of limit cycles through supercritical Hopf bifurcations can be easily identified and interpreted. In contrast, the HH model provides a biophysically detailed representation of neuronal activity, incorporating multiple ionic currents and gate opening variables.

This higher-dimensional nonlinear system (four dynamic variables) introduces significantly richer dynamics, including more complex transient responses, realistic spike generation, and a more intricate bifurcation structure. Unlike the FHN model, where bifurcation behavior is largely symmetric and smooth, the HH system exhibits both subcritical and supercritical Hopf bifurcations, indicating abrupt transitions between stable and oscillatory regimes, as is mentioned by other studies (Dimitrichev, 2018). This reflects the underlying nonlinear coupling between ion channel kinetics and membrane potential, which cannot be captured by simplified models. From a computational perspective, the simplicity of the FHN model allows for efficient numerical integration and rapid exploration of parameter space, making it particularly suitable for bifurcation studies and theoretical analysis. The HH model, on the other hand, requires more advanced numerical methods and careful step-size control due to stiffness and nonlinear coupling, as reflected in the superior performance of adaptive solvers such as ODE45.

Despite these differences, both models consistently demonstrate that the external current acts as a critical control parameter governing the transition between dynamical regimes. The presence of Hopf bifurcations in both systems confirms that neuronal oscillations arise as a fundamental nonlinear phenomenon, independent of model complexity, although the qualitative and quantitative characteristics of these transitions differ significantly.

Conclusions

The phenomenon of neuronal potential oscillation exhibits interesting behaviour, the dynamics of which consists of fixed points, oscillations and Hopf bifurcations depending on the control parameter, which in this case is the current I. The presented simple mathematical models give us acceptable information on phenomenon previously experimented in real cases. Analytical studies of differential equations can be replaced by numerical integrations and computer simulations from which the results indicate that simplicity and acceleration of calculations can be achieved with advanced methods for solving differential equations. The use of the ODE45 and ODE23 package, part of the MATLAB program, facilitates the work of performing the necessary simulations.

From the simplified case FN we observe that the system exhibits two supercritical Hopf bifurcations for different values of the current I, where between the two bifurcations the system follows a limit cycle like the relaxation oscillatory models. In HH model the system exhibits two Hopf bifurcations, one subcritical and one supercritical Hopf Bifurcation for different values of the current I. In the most advanced model, we have more information on the system, such as channel currents, channel opening probabilities, etc. compared to modelling with a simple electric circuit. We also notice that in this case we have two Hopf bifurcations where the system changes stability, from a stable fixed point to a stable oscillation according to a limit cycle, and again to a stable fixed point but now for a higher value of the potential which is known as the potential of neuron excitation.

References

A. L Hodgkin, A F Huxley. (1945). Resting and action potentials in single nerve fibres. J. Physiol.

A. L Hodgkin, A F Huxley. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. J Physiol.

https://doi.org/10.1113/jphysiol.1952.sp004764

A. L. Hodgkin, A. F. Huxley. (1951). The components of membrane conductance in the giant axon of loligo. Physiol.

Balth. Van der Pol, J. Van der Mark. (1928). The Heartbeat considered as a Relaxation Oscillation and an Electrical Model of the Heart.

Bisquert, J. (2021). A Frequency Domain Analysis of the Excitability and Bifurcations of the FitzHugh−Nagumo Neuron Model. https://doi.org//10.1021/acs.jpclett.1c03406

Boriçi, A. (2020). Analiza numerike.

Brown, A. M. (2010). A companion guide to the Hodgkin-Huxley papers. The Journal of Psycology.

C Morris, H Lecar. (1981). Voltage oscillations in the barnacle giant muscle fiber. Biophysics journal. https://doi.org/10.1016/S0006-3495(81)84782-0

Dimitrichev, A. S. (2018). Nonlinear dynamical models of neurons, review.

Edoardo Centofanti, Massimiliano Ghiotto, Luca F. Pavarino. (2024). Learning the Hodgkin–Huxley model with operator learning techniques. Computer Methods in Applied Mechanics and Engineering. https://doi.org/10.1016/j.cma.2024.117381

Ervis Gega, Dode Prenga. (2024). Dynamics and Bifurcation in Fibre Fuse Phenomenon. European Modern Studies Journal, 8, 350 - 366. https://doi.org/10.59573/emsj.8(5).2024.30

Ervis Gega, Fotion Mitrushi, Jurgen Shano, Ramadan Firanj. (2024). The study of dynamics and quazi zero stiffnes vibration isolator withn numerical integration method. Buletins of Natural science.

FitzHugh. (1961). Impulses and Physiological States in Theoretical Models of Nerve Membrane. Biophysical Journal. https://doi.org//10.1016/S0006-3495(61)86902-6

Giovannini, F. (2017). Mathematical Modelling of Neural Oscillations in Hippocampal Memory Networks during Waking and under General Anaesthesia.

Hafeez Y Hafeez, Chifu E. Ndikilar. (2014). Van der Pol Equation for Nonlinear Plasma Oscillations. American Scientific Publishers, 1-4.

HODGKIN, A. L. (1937). Evidence for electrical Transmison In Nerve.

Hu Wang, Sha Wang, Yajuan Gu, Yongguang Yu. (2023). Hopf Bifurcation Analysis of a Two-Dimensional Simplified Hodgkin–Huxley Model. https://doi.org//10.3390/math11030717

Hubert Eichner, Alexander Borst. (2011). Hands-On Parameter Search for Neural Simulations by a MIDI-Controller. https://doi.org/10.1371/journal.pone.0027013

Izhikevich, E. M. (2007). Dynamical Systems in Neuroscience.

J. E. Marsden, M. McCracken. (1976). The Hopf Bifurcations and its Applications.

Kenneth Leander Anderson Jr., Jackie Chism, Quarail Hale, Paul Klockenkemper, Chelsi Pinkett, Christopher Smith, Dr. Dorjsuren Badamdorj. (2013). Mathematical Modeling Action Potential in Cell Processes.

Kznetsov, Y. (1998). Elements of Applied Bifurcation Theory. Amsterdam: Springer.

Matilde Marcolli and Doris Tsao. (2017). The Neuron as a Dynamical System.

Maurice Courbage, Vladimir I. Nekorkin, Lev V. Vdovin. (2007). Chaotic oscillations in a map-based model of neural activity. HAL. https://doi.org/10.1063/1.2795435

Nagumo, J., Arimoto, S.Yoshitzawa, S. (1962). Dynamics of Nerve Pulse Propagation in a Weakly Dissipative Myelinated Axon. https://doi.org//10.1109/JRPROC.1962.288235

Pascal Wallisch, Michael Lusignan, Marc Benayoun, Tanya I. Baker, Adam Dickey, Nicholas G. Hatsopoulos,. (2014). Matlab For Neuroscientist.

Rose T. Faghih, Ketan Savla, Munther A. Dahleh, Emery N. Brown. (2010). The FitzHugh-Nagumo Model: Firing Modes with Time-varying Parameters & Parameter Estimation. https://doi.org/10.1109/IEMBS.2010.5627326

Ruru Wang, Yanping Chen, Leijie Qiao, Jian Huang. (2025). Numerical methods for studying neuronal dynamics in the stochastic fractional Hodgkin-Huxley model. Physics Letters A. https://doi.org/10.1016/j.physleta.2025.130966

Siciliano, R. (2012). The Hodgkin-Huxley Model.

Strogatz, S. (2015). Nonlinear Dynamics and Chaos.

William Aristizabal Botero, Alvaro H. Salas, Silvia Janeth Gonzalez. (2012). The Hodgkin-Huxley neuron model on the fast phase plane. International Journal of Physical Sciences.

Xue Zhong, Yaze Liu, Tumurpurev Namnan, Hexi Baoyin. (2025). Dynamical mechanism of neuronal firing in the Hodgkin-Huxley model. Research Square.

https://doi.org/10.21203/rs.3.rs-7322686/v1