OPTIMIZATION OF HISTOGRAMS IN HYDROLOGICAL TIME SERIES ANALYSIS: A COMPARATIVE DATA-DRIVEN APPROACH

RUDIN HOXHA

Sector of Statistics, Models and Indicator Analysis,

National Cyber Security Authority, Tirana, Albania

e-mail: rudinhocha@gmail.com

Abstract

The primary objective of this study is the application of advanced statistical methods, based on empirical data, to design optimal histograms for the water inflows of the three hydropower plants of the Drin Cascade. The focus of this paper is not on identifying a universally superior method, but on selecting the most appropriate technique depending on the specific characteristics of the data and the physical phenomena of each basin. For Fierza and Koman, Knuth's method proved to be efficient, producing optimal histograms that are statistically accurate and easily interpretable. In contrast, for Vau i Dejës, where the data distribution presents high complexity, the adaptive Bayesian Blocks method emerged as the appropriate solution to capture details that standard methods fail to highlight. Based on the optimal histograms, Probability Density Functions (PDFs) and Cumulative Distribution Functions (CDFs) were constructed for each basin using the Piecewise Linear approach. For seasonal trend analysis, Nadaraya-Watson Kernel Regression with periodic wrapping was applied, enabling the construction of annual hydrographs that clearly identify the nival and pluvial regimes of the cascade.

Key words**:** *Hydrology, Drin Cascade, histogram optimization, Bayesian blocks, kernel regression.*

Përmbledhje

Objektivi kryesor i këtij studimi është aplikimi i metodave të avancuara statistikore, bazuar në të dhëna empirike, për të hartuar histograma optimale për prurjet e ujit të tre hidrocentraleve të Kaskadës së Drinit. Fokusi i këtij punimi nuk është identifikimi i një metode universalisht superiore, por në përzgjedhjen e teknikës më të përshtatshme në varësi të karakteristikave *specifike të të dhënave dhe fenomeneve fizike të secilit pellg. Për Fierzën dhe Komanin, metoda e Knuth-it rezultoi efikase, duke prodhuar histograma optimale që janë statistikisht të sakta dhe lehtësisht të interpretueshme. Në të kundërt, për Vaun e Dejës, ku shpërndarja e të dhënave paraqet kompleksitet të lartë, metoda adaptive e Blloqeve Bayesian doli si zgjidhja e duhur për të kapur detajet që metodat standarde nuk arrijnë t'i nxjerrin në pah. Bazuar në histogramet optimale, Funksionet e Dendësisë së Probabilitetit (PDF) dhe Funksionet e Shpërndarjes Kumulative (CDF) u ndërtuan për secilin pellg duke përdorur qasjen Piecewise Linear. Për analizën e trendit sezonal, u aplikua Regresioni i Bërthamës Nadaraya-Watson me mbështjellje periodike, duke mundësuar ndërtimin e hidrografeve vjetore që identifikojnë qartë regjimet nivale dhe pluviale të kaskadës*

Fjalë kyçe: *Hidrologjia, kaskada e Drinit, optimizimi i histogramit, blloqet Bayesian-e, regresioni i bërthamës.*

Introduction

The Drin River Cascade represents one of the most critical infrastructures for energy production and water resource management in the Balkan region. The accurate statistical modeling of lateral inflows into the three main hydropower plants (HPP)—Fierzë, Koman, and Vau i Dejës—is essential for optimizing reservoir operations, assessing flood risks, and ensuring sustainable energy generation. However, hydrological time series often exhibit complex behaviors, including multimodal distributions, heavy tails representing extreme events, and seasonal variabilities that challenge traditional statistical methods. Previous studies have established that the distributions of average hourly and daily inflows are non-stationary, at least over observation periods of up to 10 years. By employing q-functions (Prenga and Ifti, 2014) and (Kovaçi et al., 2015) have determined that these distributions deviate significantly from the reference log-normal distributions. Furthermore, the non-stationarity parameter, known as q-stationarity, yields a value of approximately 2, whereas stationary conditions require $1 \leq q \leq 5/3$, as per (Umarov et al., 2008).

Similarly, (Porja, Prenga., 2024) identified the same characteristics in comparable hydrological time series within our country, alongside considerable deviations from the expected Gumbel distribution. We note that a more meticulous approach to characterizing these distributions could significantly improve the identification of these underlying patterns.

Consequently, we emphasize the necessity of re-evaluating the discretization process as a preliminary step, as well as refining regression models wherever possible. A comparable strategy has proven successful in analyzing the time series of our systems under similar conditions (Prenga et al., 2025). Standard approaches to data analysis, particularly the construction of histograms and Probability Density Functions (PDFs), often rely on empirical rules (such as Sturges' or Scott's rules) that assume underlying normality. In the context of the Drin Cascade, where inflow data is characterized by measurement artifacts and distinct physical regimes (nival versus pluvial), these "one-size-fits-all" methods frequently fail to capture the true underlying structure of the data. This study addresses these challenges by applying advanced, data-driven statistical techniques. The objective is to design optimal histograms and continuous probability functions that accurately reflect the physical phenomena of each basin without imposing arbitrary parametric assumptions. By comparing fixed-width binning optimization against adaptive segmentation methods, this paper seeks to provide a robust methodological framework for hydrological time series analysis.

Literature review

The statistical analysis of hydrological data fundamentally relies on the accurate estimation of probability density functions. While the histogram is the oldest and most widely used density estimator, its accuracy is heavily dependent on the selection of bin width. (Knuth, 2006) revolutionized this approach by introducing a Bayesian framework for finding the optimal bin width that maximizes the posterior probability of the model, treating the histogram as a piecewise-constant model. However, Knuth also highlighted the challenge of "digitized data"—measurements recorded as integers—which can cause such algorithms to fail by creating infinite density spikes.

In cases where data density varies significantly across the domain, fixed-width histograms may result in loss of information. (Scargle et al., 2013) proposed the Bayesian Blocks method, a non-parametric technique originally developed for astronomical time series. This method uses dynamic programming to segment data into blocks of variable widths, allowing for high resolution in high-density areas and stability in low-density tails (Pollack et al., 2017).

Furthermore, the transition from discrete histograms to continuous functions requires careful handling to avoid boundary bias, a common issue in Kernel Density Estimation (KDE) for non-negative variables like river flows

(Baszczyńska, 2016). For temporal analysis, non-parametric regression techniques, such as the Nadaraya-Watson estimator, offer flexibility in modeling seasonal trends without assuming a specific functional form (Rajagopalan et al., 1997). Recent reviews by (Haddad, 2025) emphasize the growing importance of such Bayesian and data-driven methods in modern hydrological modeling to capture uncertainty and complex dynamics.

Methodology

This study is based on the statistical analysis of time series of lateral inflows for the three main hydropower plants (HPP) of the Drin Cascade: Fierzë, Koman, and Vau i Dejës. The database presents variations in the length of time series according to the availability of measurements. Specifically, for Lake Fierza, the series consists of 12,570 daily data points covering the period January 1, 1991 – May 31, 2025. For the lakes of Koman and Vau i Dejës, the series contain 10,010 data points each, covering the period January 1, 1998 – May 31, 2025. This extensive database offers the opportunity for a comprehensive and reliable assessment of hydrological regimes.

Before applying advanced probability density estimation algorithms, the data underwent a rigorous cleaning and standardization process. This step was critical to eliminate numerical artifacts and adapt the time series to the natural hydrological cycle.

Treatment of Digitized Data

A detailed analysis of the raw data revealed an inconsistency in measurement precision over the years. While recent data are recorded with decimal precision (float), early records often appear as integers. This discrete nature of historical data corresponds to the problem of "digitized data" described by (Knuth, 2006). This creates fundamental problems for histogram optimization algorithms. Knuth's method, which relies on calculating posterior probability, fails to converge in the presence of these discrete "shelves," as it interprets them as points with infinite probability density.

To solve this problem without compromising the engineering integrity of the measurements, the "Jittering" (White Noise Injection) technique was applied. A microscopic uniform noise component ϵ was added to each measured value Q_{raw}, following the distribution:

$$Q_{processed} = Q_{raw} + \epsilon, \quad \text{where } \epsilon \sim U[-0.005, 0.005]$$

This process transforms the distribution from discrete to mathematically continuous. The absolute change in value is less than $0.01\mathrm{m}^3/\mathrm{s}$—a negligible magnitude for hydro-technical analyses, but sufficient to stabilize log-posterior functions during bin optimization.

Indexing by Hydrological Year

After adjusting numerical values, the next step was temporal adaptation. Traditional analysis based on the calendar year (Jan 1 – Dec 31) presents limitations for hydrological studies in the Balkan region, as it artificially splits the wet season into two different years. To maintain the continuity of the precipitation cycle and accurately model seasonal trends, all data were re-indexed according to the Hydrological Year, defined from October 1st to September 30th.

In this scheme, October 1st corresponds to the daily index $t = 0$, while September 30th corresponds to $t = 364$ (or 365 in leap years). This transformation ensures that the expected peak of winter and spring flows is positioned in the center of the analysis window, avoiding edge effects at the boundaries of the time series during the application of smoothing techniques.

Analysis and discussion

Probability density estimation via Knuth’s method

Empirical estimation of the Probability Density Function (PDF) is the fundamental step in the statistical modeling of water inflows. One of the main challenges in constructing empirical histograms is selecting the number of intervals or "bins" (M). Traditional empirical rules, such as Sturges’ or Scott’s rule, often fail in the case of multimodal distributions or those with high asymmetry, as they assume a normal distribution of data, which rarely occurs in hydrology.

To address this problem, this study applied the optBINS method developed by Kevin Knuth in 2006. This method is based on Bayesian inference and treats the histogram as a "piecewise-constant model." The goal is to find the number of intervals M that maximizes the posterior probability $P(M|D)$, based solely on the observed data D. The method automatically balances two opposing forces: the Likelihood Function (which increases with the addition of bins) and the Prior Probability (which penalizes complex models).

Flow analysis for Lake Fierza

The time series of Fierza ($N = 12{,}570$) presents a unique challenge due to its highly skewed nature and the "heavy tail" representing rare flood events. Applying Knuth's method to the Fierza data resulted in an optimal number of bins $M = 104$. The analysis of the Log-Posterior function showed a clear and smooth maximization at this value, confirming the success of the preliminary digitized data treatment process.

It is worth noting that, to verify the stability of this result, the algorithm's search space was expanded up to $M = 5{,}000$ bins. The results showed that the optimum initially achieved at $M = 104$ remained unchanged and represents a stable global maximum, excluding the possibility of local maxima. This relatively high number of intervals is statistically justified by the need to model two different regimes within the same distribution: the Low/Medium Flow Regime where data density is very high, and the Extreme Flow Regime where the distribution extends to values over $2000\text{m}^3/\text{s}$.

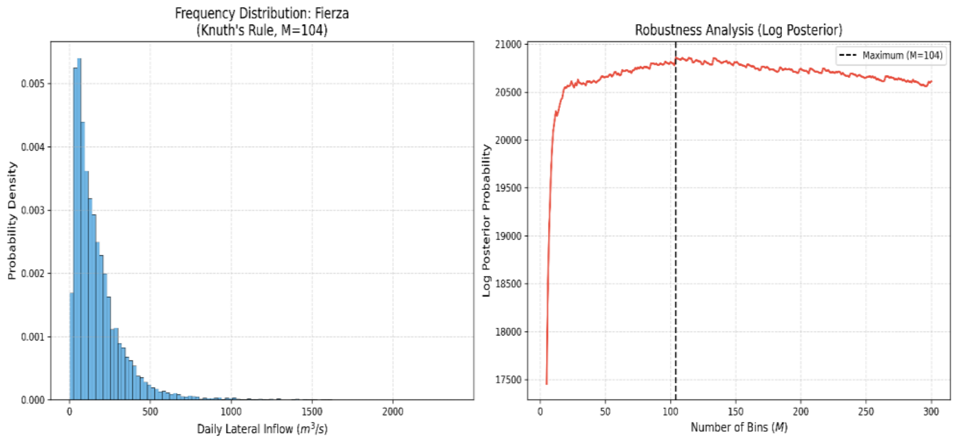

Figure 1. Optimal histogram according to Knuth's method for HPP Fierza. The left graph shows probability density, while the right graph presents the Log-Posterior function.

Analysis for HPP Koman

Time series analysis for HPP Koman ($N = 10{,}010$) revealed an even more complex dynamic. Application of the optBINS algorithm resulted in an optimal number of bins $M = 188$. Similar to the Fierza case, an extended test was performed for Koman by examining log-posterior behavior up to 5,000 bins. This verification confirmed that the value $M = 188$ constitutes the global optimum of the model.

This result ($M = 188$) is significantly higher than that of Fierza ($M = 104$), despite the Koman time series being shorter. From a statistical perspective, this indicates that the distribution of lateral inflows in Koman is characterized by more pronounced local variation. The histogram displays an extremely "sharp peak" in the low flow zone and a rapid decline towards average values.

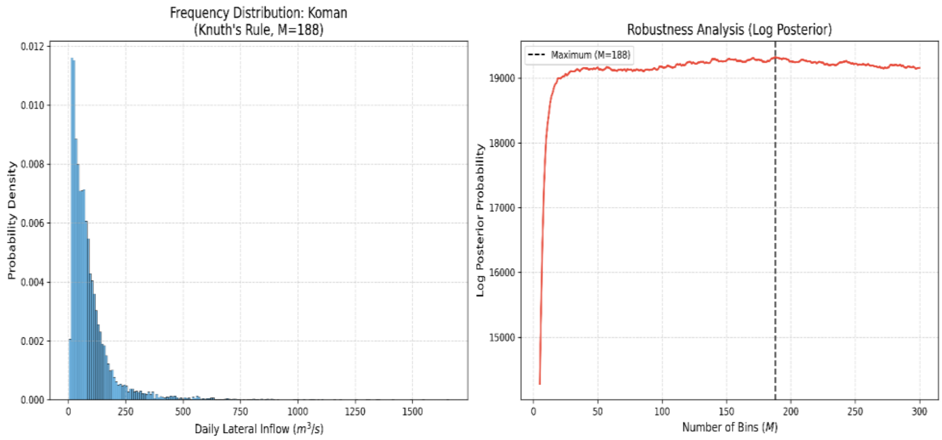

Figure 2. Optimal histogram according to Knuth's method for HPP Koman.

Convergence failure for HPP Vau i Dejës

For HPP Vau i Dejës, the application of Knuth's method revealed fundamentally different characteristics of the time series compared to the other two basins. Unlike Fierza and Koman, where the log-posterior function showed a clear global maximum, for Vau i Dejës the algorithm failed to converge. Even after expanding the search space to $M = 5{,}000$ bins, the log-posterior function continued to exhibit asymptotic growth, suggesting an unstable optimum around the value $M \approx 4973$.

This behavior indicates that the data structure is highly fragmented or contains strong discretization artifacts that dominate over the general shape of the distribution (PDF). The algorithm attempts to "model the noise" by creating an extreme number of intervals to fit microscopic data fluctuations. This failure dictated the need to switch to more advanced methods with variable interval widths, specifically the Bayesian Blocks method.

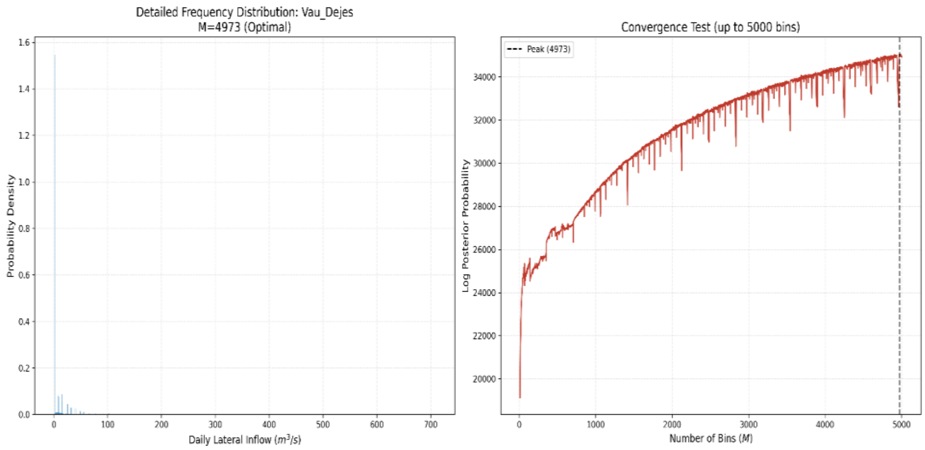

Figure 3. Analysis of Knuth's Method for HPP Vau i Dejës. The graph on the right shows the continuous increase of the log-posterior without a clear peak.

Probability estimation via Bayesian Blocks method

The failure of fixed-width histograms (Knuth's method) to converge to an optimal solution for Vau i Dejës signals the presence of a complex data structure. To address this issue, this study applies the adaptive segmentation method known as Bayesian Blocks, proposed by (Scargle et al. 2013). Unlike classical histograms, the Bayesian Blocks method allows the width of each "block" (bin) to be dynamically determined by the data itself.

Theoretical principles of adaptive segmentation

The algorithm is based on a non-parametric approach that uses dynamic programming to find the exact partition (globally optimal partition) that maximizes a fitness function. For event data, the fitness function for a single block k is given by:

$$\log L^{(k)} = N^{(k)} \log \left(\frac{N^{(k)}}{T^{(k)}} \right)$$

where $N^{(k)}$ is the number of data points in block k and $T^{(k)}$ is the width of the block. To avoid overfitting, the method incorporates a geometric prior for the number of blocks, controlled by the parameter ncp_prior, which is calibrated based on the number of data points N and an acceptable false alarm rate (p_0).

Application in HPP Vau i Dejës

Implementation of the Bayesian Blocks algorithm on the Vau i Dejës data resulted in the identification of an optimal structure consisting of $N_{blocks} = 192$. This result is in stark contrast to the thousands of bins suggested by Knuth's method and represents a much more robust statistical model.

The main advantage of this approach is observed in its ability to change the resolution of analysis. In the low peak zone ($0 - 50\text{m}^3/\text{s}$), the algorithm creates very narrow blocks, accurately capturing rapid variations in probability density. In the distribution tail ($> 200\text{m}^3/\text{s}$), the blocks expand significantly, grouping rare data to maintain statistical significance and avoid noise.

To assess the stability of this model, a Bootstrap analysis with 50 iterations was performed. Results showed high model stability, with an average number of blocks of ≈ 190 and minimal standard deviation.

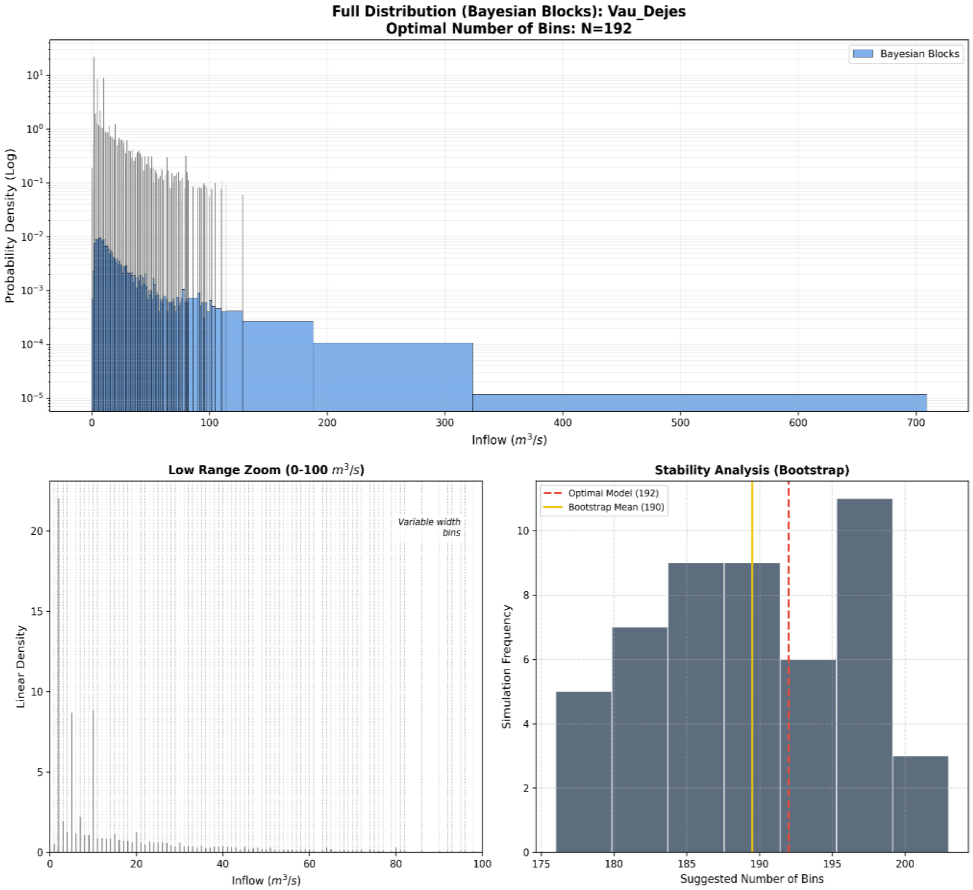

Figure 4. Bayesian Blocks results for HPP Vau i Dejës. The lower-left panel shows the magnification of the peak area.

Comparative application for Fierza and Koman

To further validate the methodology and provide a more detailed overview of the data structure, the Bayesian Blocks method was also applied to the lakes of Fierza and Koman. For HPP Fierza, the algorithm identified an optimal number of $N_{blocks} = 638$, significantly higher than $M = 104$ suggested by Knuth's method. This added resolution allows for the identification of microstructures in the flow distribution that fixed-interval methods tend to smooth out. Stability analysis via Bootstrap ($N_{boot} = 50$) confirmed the stability of this complex model.

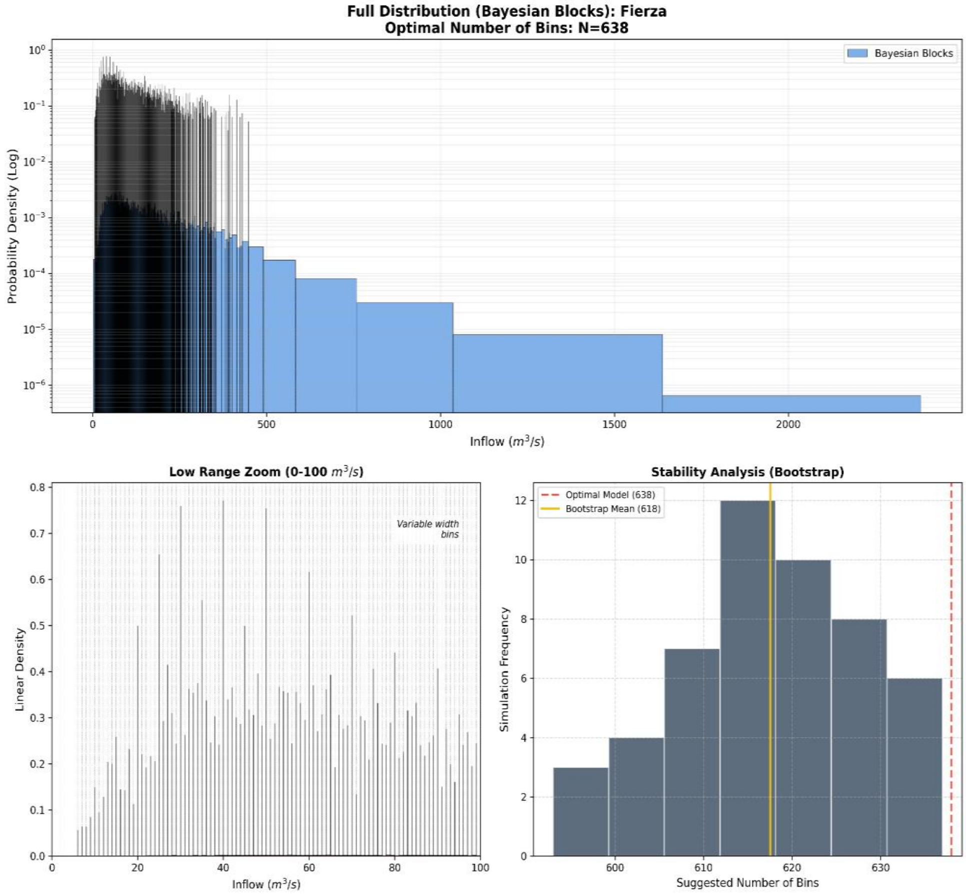

Figure 5. Bayesian Blocks results for HPP Fierza.

For HPP Koman, the analysis produced an optimal number of blocks of $N_{blocks} = 339$. Here too, this number is higher than that suggested by Knuth's method ($M = 188$), indicating higher sensitivity to local flow variations.

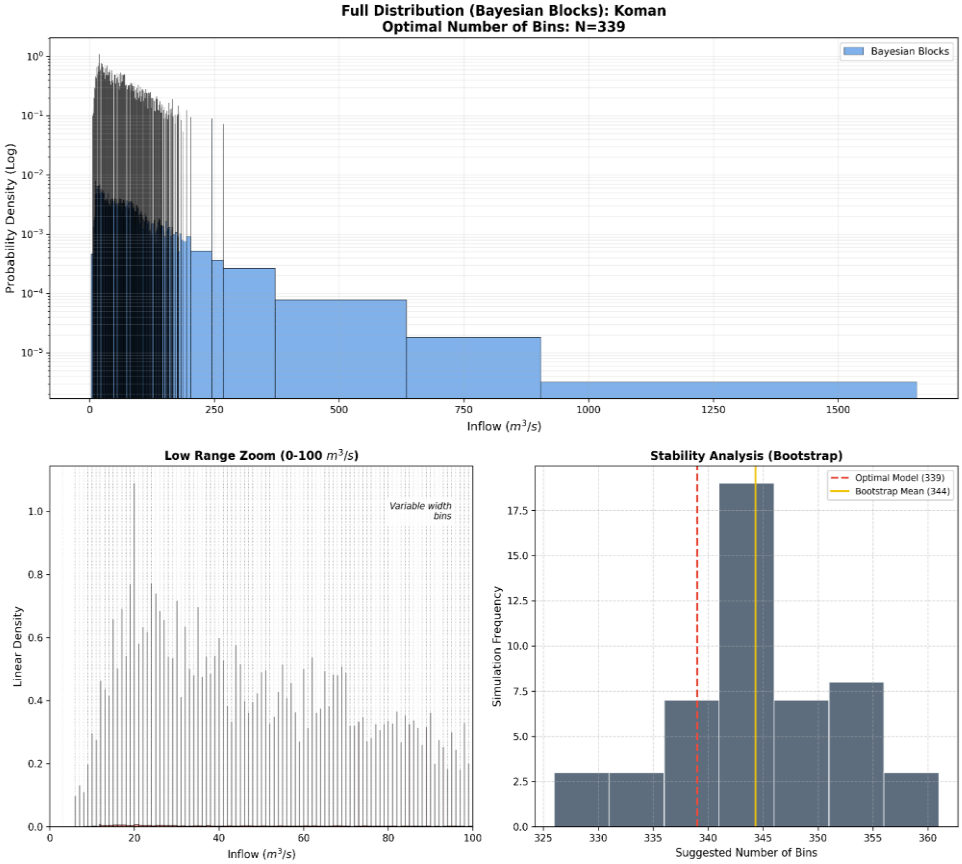

Figure 6. Bayesian Blocks results for HPP Koman.

Derivation of probability density and cumulative distribution functions

After determining the optimal histogram structure, the final engineering step is transforming these discrete structures into continuous probability functions. For PDF and CDF estimation, this study adopts a direct data-driven approach, using optimized histograms as a baseline.

Methodology of the Piecewise Linear Approach

Let n_i be the number of data points in block i and w_i the width of this block (where $i = 1, \ldots, M$). The percentage of data in block i is given as $p_i = n_i/N$, where N is the total number of data points. The Probability Density Function (PDF) is defined according to Scargle et al. (2013) as:

$$f(x) = \frac{p_i}{w_i}, \quad \text{for block } i$$

The Cumulative Distribution Function (CDF) is constructed by integrating the PDF, resulting in a piecewise linear function [7]:

$$F(x) = \sum_{j=1}^{k-1} p_j + \frac{x - x_{start,k}}{w_k} \cdot p_k$$

where $x_{start,k}$ is the left boundary of block k. This approach ensures that $F(x)$ is continuous and monotonically increasing from 0 to 1.

For the Fierza and Koman hydropower plants, PDFs were generated by normalizing the histograms obtained from Knuth's method. Normalization ensures that the total area under the curve equals 1. The corresponding CDFs were calculated as the cumulative sum of these densities, linearly interpolated between the centers of the intervals (bins).

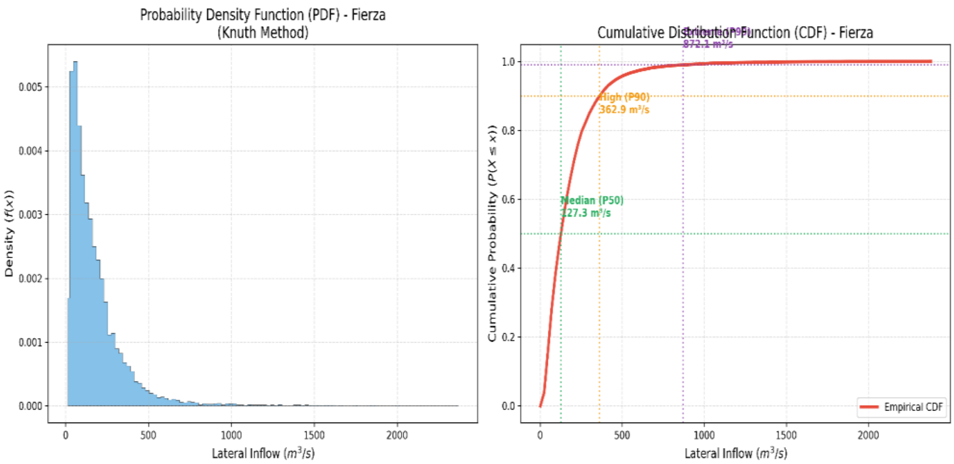

Figure 7. Graphical representation of probability density (left) and cumulative distribution (right) for HPP Fierza.

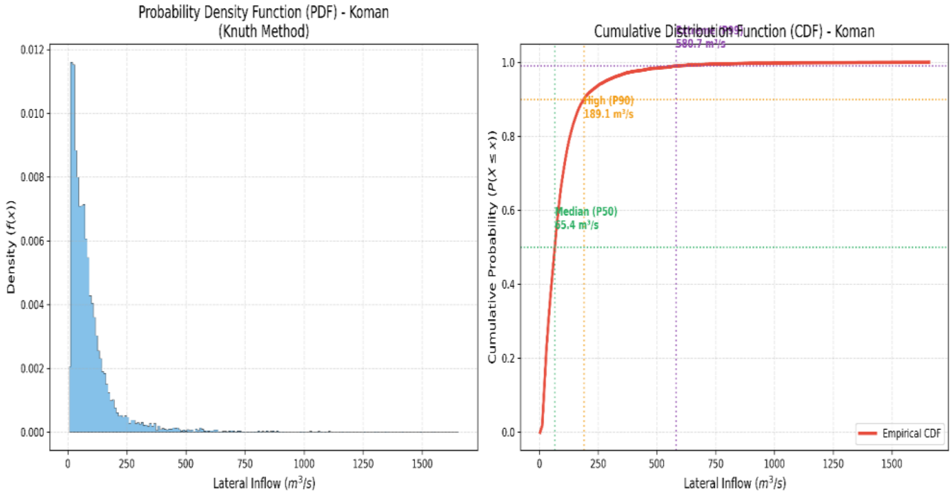

Figure 8. Graphical representation of probability density (left) and cumulative distribution (right) for HPP Koman.

For Vau i Dejës, due to the unique nature of the data, the variable block structure (Bayesian Blocks) was used to construct the PDF. The CDF derived from this model offers the highest possible accuracy for this basin, avoiding errors that would arise from assuming a uniform distribution or using standard parametric methods.

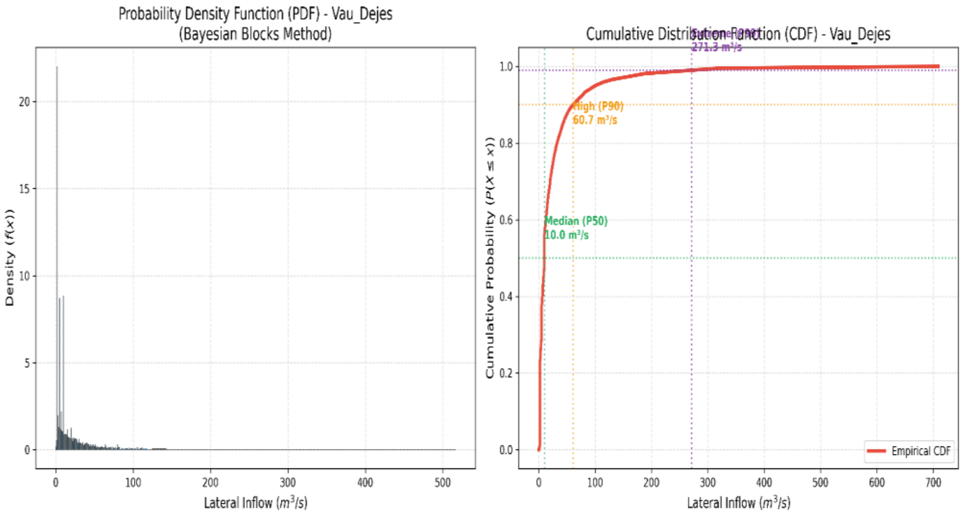

Figure 9. Probability functions for HPP Vau i Dejës using Bayesian Blocks.

Construction of annual hydrographs

This section addresses the estimation of the conditional expectation of flow depending on the day of the hydrological year t. Given that the relationship between time and flows is non-linear, a non-parametric approach based on Kernel regression was used.

Methodology of non-parametric Nadaraya-Watson Regression

To estimate the trend function $m(t)$, the Nadaraya-Watson (NW) estimator was applied, which calculates a locally weighted average of observed data. For a given evaluation day t, the estimator is given by the formula:

$$\hat{m}_h(t) = \frac{\sum_{i=1}^{n} K_h\,(t - t_i)Y_i}{\sum_{i=1}^{n} K_h\,(t - t_i)}$$

Where:

- Y_i are the observed flows (pre-treated with jittering).
- t_i are the corresponding days of the hydrological year ($0 \leq t_i \leq 365$).
- $K_h(u) = \frac{1}{h}K(\frac{u}{h})$ is the Kernel function scaled by the bandwidth parameter h.

In this study, the Gaussian Kernel was used: $K(u) = \frac{1}{\sqrt{2\pi}}e^{-0.5u^2}$.

Handling boundary effects: Periodic Wrapping

A standard problem in kernel regression is "boundary bias." In the context of the hydrological cycle, day $t = 0$ (Oct 1) and day $t = 365$ (Sep 30) are physically continuous but mathematically distant on a linear axis. To ensure that the smoothed curve is cyclic and smooth at the year transition point, the Periodic Wrapping technique was implemented by expanding the dataset D with "phantom copies" of data:

$$D_{aug} = \{(t_i - 365, Y_i)|t_i \in [305{,}365]\} \cup D \cup \{(t_i + 365, Y_i)|t_i \in [0{,}60]\}$$

Bandwidth selection via LSCV

The bandwidth parameter h controls the degree of smoothing. To select optimal h objectively, the Least Squares Cross-Validation (LSCV) method was used. Considering the high autocorrelation of hydrological time series, the "Leave-One-Year-Out" scheme was applied:

$$LSCV(h) = \frac{1}{N_{years}} \sum_{k=1}^{N_{years}} \frac{1}{n_k} \sum_{i \in Year_k} (Y_i - \widehat{m}_{h,-k}(t_i))^2$$

Where $\widehat{m}_{h,-k}$ is the estimator trained with all years except year k.

Results by basin

Seasonal trend analysis for HPP Fierza clearly reveals a Nival hydrological regime (dominated by snowmelt). The graph shows that flows begin to increase in late winter, reaching a maximum peak in May. The 95% confidence intervals (yellow zone) are narrower during summer, indicating high predictability of minimum flows.

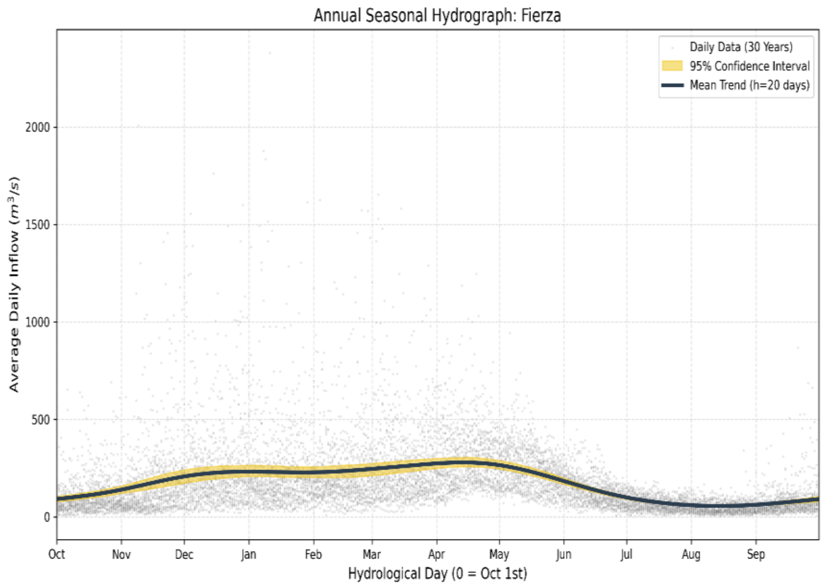

Figure 10. Smoothed annual hydrograph for HPP Fierza using Nadaraya-Watson regression (h = 20 days).

Analysis for HPP Koman exhibits different behavior, with a clearer Pluvial dominance (related to rainfall). The flow peak is more distributed during winter and early spring, reflecting the combined influence of direct precipitation and discharges from Fierza.

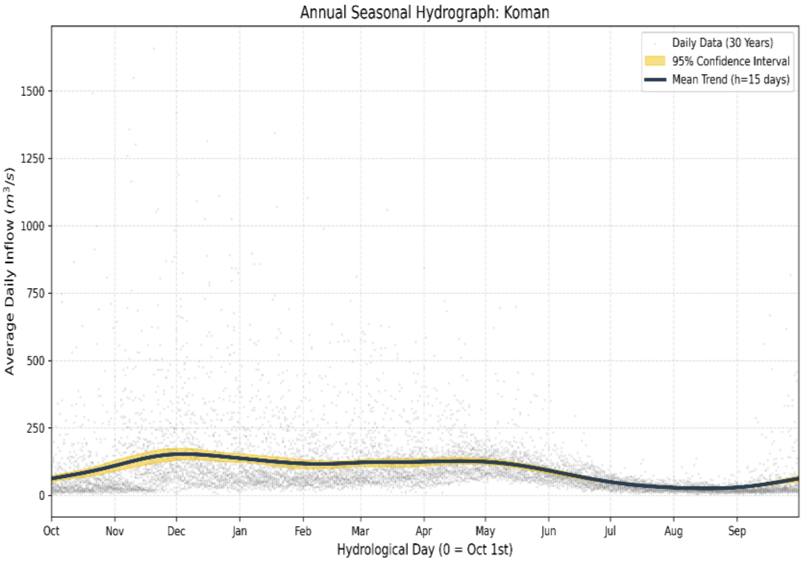

Figure 11. Smoothed annual hydrograph for HPP Koman using Nadaraya-Watson regression (h = 15 days).

HPP Vau i Dejës shows a profile similar to Koman, where the flow peak is concentrated in the winter period (December-March), indicating a typical Pluvial regime. However, the trend line is flatter compared to the two upper basins, which can be explained by the regulatory effect of the upper lakes and local geology.

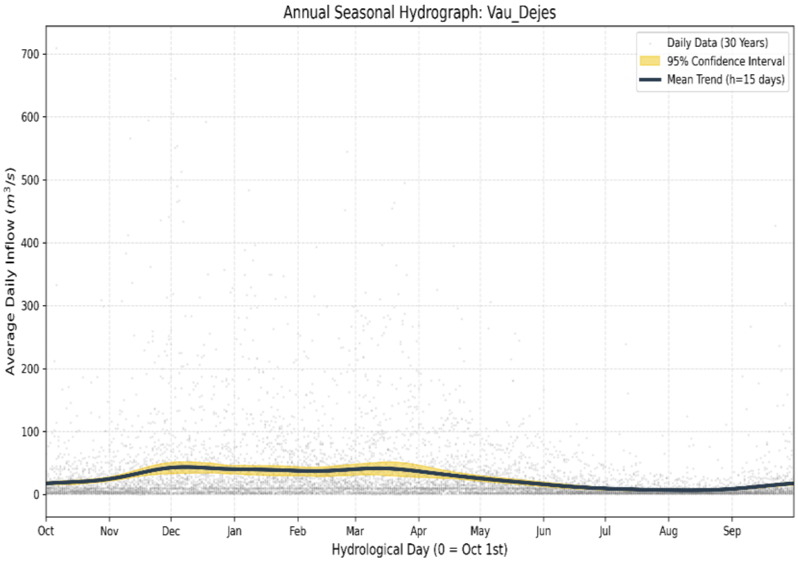

Figure 12. Smoothed annual hydrograph for HPP Vau i Dejës using Nadaraya-Watson regression (h = 15 days).

Conclusion

This study offers an in-depth statistical analysis of the hydrological regime of the Drin Cascade, applying modern non-parametric methods for characterizing lateral inflows in the three main hydropower plants: Fierzë, Koman, and Vau i Dejës. The use of advanced algorithms such as Knuth's method, Bayesian Blocks, and Nadaraya-Watson Regression has enabled an objective and detailed assessment of probability distributions and seasonal trends, overcoming the limitations of traditional methods.

The main findings of this paper can be summarized as follows:

- **Efficiency of Bayesian Methods in Handling Complex Data:** The application of Knuth's method proved successful for Fierza and Koman, identifying the optimal histogram structure. However, its failure in the case of Vau i Dejës and the subsequent success of the Bayesian Blocks method demonstrates the importance of adaptability in statistical modeling. The Bayesian Blocks method proved superior in capturing local variations and complex data structures, especially in areas with high information density and in the distribution tail.

•

- **Clear Differentiation of hydrological regimes:** Temporal analysis via Nadaraya-Watson regression confirmed the existence of two different regimes in the cascade. Fierza exhibits a Nival regime with a peak in spring (May), influenced by snowmelt, while Koman and Vau i Dejës follow a Pluvial regime with a peak in winter, dominated by rainfall. This differentiation is critical for optimized management of water reserves.
- **Advantages of the Non-Parametric Approach:** The use of Piecewise Linear functions for PDF and CDF, as well as kernel smoothing for annual hydrographs, eliminated the need for a priori assumptions about the shape of distributions. This "data-driven" approach ensured that results faithfully reflect the nature of the data, avoiding systematic errors (bias) that often accompany parametric methods or standard KDE at domain boundaries.
- **Uncertainty assessment:** The integration of Bootstrap techniques and cross-validation (LSCV) offered a quantitative measure of result reliability. The 95% confidence intervals for annual hydrographs clearly show that uncertainty is higher during wet periods, key information for flood risk assessment.

Furthermore, these findings and the application of similar methodologies—particularly the Bootstrap technique—are anticipated to enhance predictive capabilities through the re-evaluation of the Empirical Cumulative Distribution Function (ECDF). This re-evaluation serves as a preliminary step for simulation-based calculations, ensuring beforehand that the dataset meets the procedural requirements of this technique, including the stationarity conditions for the target distribution in Monte Carlo (MC) simulations. Addressing this objective will be the primary focus of our forthcoming research.

In conclusion, this paper demonstrates that the application of advanced statistical methods in hydrology not only improves estimation accuracy but also offers deeper insight into the physical dynamics of water systems. It is recommended that future studies explore the integration of these methods into operational predictive models and analyze the impact of climate change on these identified regimes

References

A. Baszczyńska. “Kernel Estimation of Cumulative Distribution Function of a Random Variable with Bounded Support,”Statistics in Transition New Series, 17(3), 541-556, (2016).

B. Pollack, S. Bhattacharya, & M. Schmitt (2017). “Bayesian Block Histogramming for High Energy Physics,” arXiv:1708.00810.

B. Rajagopalan, U. Lall, & D.G. Tarboton (1997). “Evaluation of kernel density estimation methods for daily precipitation resampling. Stochastic Hydrology and Hydraulics,” 11, 523-547.

D. Freedman, & P.Diaconis. (1981). “On the histogram as a density estimator: L2 theory,”Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete, 57, 453-476.

D. Prenga, & M. Ifti. (September 2014) “Complex System Approach to the Study of Albanian Drin River Inflows”, Hydrological Sciences Journal.

K. Haddad (2025). “A Comprehensive Review and Application of Bayesian Methods in Hydrological Modelling: Past, Present, and Future Directions,”Water, 17, 1095.

K. H. Knuth (2006). “Optimal Data-Based Binning for Histograms,”arXiv:physics/0605197.

J. D. Scargle, J. P. Norris, B. Jackson, & J. Chiang (2013). “Studies in Astronomical Time Series Analysis. VI. Bayesian Block Representations,” The Astrophysical Journal, 764(2), 167.

S. Kovaçi, D. Prenga, & M. Ifti (2015). “Analysis of distributions and dynamics for water side inflows in the Drin River basin, Albania”. International Journal of Engineering and Technical Research.

Umarov, S., Tsallis, C., & Steinberg, S. (2008). On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics. Milan Journal of Mathematics, 76, 307–328.

T. Porja, &D. Prenga (2024), “From Wet to Extreme Wet Spells -Case Study of Shkodra”, ECES 2024.

X. Chen, J. Parajka, B. Széles, P. Valent, A. Viglione, &G. Blöschl. (2020). “Impact of Climate and Geology on Event Runoff Characteristics at the Regional Scale.”Water, 12, 3457.